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# TRANSLATION AND COMMENTARY OF CHAPTER SIX OF QUṬB AL-DĪN AL-SHĪRĀZĪ'S *NIHĀYAT AL-IDRĀK FĪ DIRĀYAT AL-AFLĀK*: ON THE ORBS AND MOTIONS OF THE SUN

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**ABSTRACT**
Quṭb ad-Dīn al-Shīrāzī (d. 1311 CE) is a famous Muslim Persian polymath who has contributed to many sciences, especially philosophy, medicine, and astronomy. For this article, we would focus on his contribution to astronomy, particularly a chapter in his well-known work, *Nihāyat al-Idrāk fī Dirāyat al-Aflāk* (The Limit of Understanding of the Knowledge of the Heavens). The chapter is the sixth chapter from the 2nd treatise of *Nihāyat al-Idrāk*, which is known as "On the Orbs and Motions of the Sun." This article is focused on translating the chapter, which will be the first half of this article, and the other half of this article, will be the commentary on the translation. Although most of the Arabic-Muslim astronomers, including Quṭb ad-Dīn al-Shīrāzī, follow the Ptolemaic astronomical paradigm, which is based on the geocentric model of the orbs instead of the heliocentric model, this does not prevent the Arabic-Muslim astronomers and including Quṭb ad-Dīn al-Shīrāzī from producing original works in astronomy. As an example, a translation of this sixth chapter of the 2nd treatise of *Nihāyat al-Idrāk* would give some evidence of Quṭb ad-Dīn al-Shīrāzī's contribution to astronomy.
**Keywords**: *Quṭb ad-Dīn al-Shīrāzī, Islamic Astronomy, Ptolemaic Astronomy, Motion of the Sun in Islamic Astronomy, Marāghah observatory*.

**ABSTRAK**
Quṭb ad-Dīn al-Shīrāzī (m. 1311 Masihi) adalah seorang sarjana polymath Muslim Parsi yang telah banyak menyumbang dalam beberapa bidang sains, terutama sekali falsafah, perubatan dan astronomi. Untuk artikel ini, kami fokuskan kepada sumbangannya di dalam bidang astronomi, terutama sekali kepada sebuah bab di dalam bukunya yang terkenal, *Nihāyat al-Idrāk fī Dirāyat al-Aflāk* (Had di dalam Memahami Ilmu Angkasaraya). Bab yang dimaksudkan adalah bab ke-enam dari buku ke-dua *Nihāyat al-Idrāk*, yang bertajuk "Tentang Orbit dan Pergerakan Matahari." Artikel ini fokus kepada menterjemahkan bab tersebut, pada bahagian pertama artikel ini, dan pada bahagian kedua artikel ini adalah komentar kepada terjemahan tersebut. Walaupun kebanyakan ahli astronomi Arab-Muslim, termasuklah Quṭb ad-Dīn al-Shīrāzī, mengikuti paradigma astronomi Ptolemy, yang menggunakan model geopusat dan bukannya model heliosentrik, ini tidak menghalang ahli astronomi Arab-Muslim, termasuklah Quṭb ad-Dīn al-Shīrāzī, dari menghasilkan hasil kajian yang asli di dalam bidang astronomi. Sebagai contoh, terjemahan bab ke-enam dari buku ke-dua *Nihāyat al-Idrāk* ini akan memberi sedikit gambaran tentang sumbangan Quṭb ad-Dīn al-Shīrāzī di dalam bidang astronomi.
**Katakunci**: *Quṭb ad-Dīn al-Shīrāzī, Astronomi Islam, Astronomi Ptolemy, Pergerakan Matahari di dalam Astronomi Islam, Balaicerap Marāghah*.

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## 1. Introduction

Quṭb ad-Dīn al-Shīrāzī (d. 1311 CE) is a famous Muslim Persian polymath who has contributed in many fields such as theology, philosophy, and illuminationist theosophy (*ishrāq*), Qur'anic commentaries, the sciences of *ḥadīth*, medicine, astronomy, mathematics, geography, physics, and even poetry. He is highly multi-talented, and his name and works are on par with those of many other great polymaths of Muslim scholars such as Ibn Sīnā, al-Fārābī, al-Bīrūnī, al-Ghazālī, Ibn Rushd, Naṣīr al-Dīn al-Ṭūsī, and many others. His main contributions, however, are usually centered on three main areas of scholarship, which are philosophy, medicine, and astronomy. In astronomy, particularly, he has contributed to and worked at the famous Marāghah observatory, which was built and founded by his teacher, the legendary Naṣīr al-Dīn al-Ṭūsī (d. 1274 CE). Here at Marāghah, Quṭb ad-Dīn al-Shīrāzī also had working relations with many other scholars that are related to the observatory, such as Mūayyad al-Dīn al-'Urḍī (d. 1266 CE), Muḥyī al-Dīn al-Maghrībī (d. 1283 CE), Kamāl al-Dīn al-Fārisī (d. 1320 CE), Bar Hebraeus (d. 1286 CE), i.e., the Christian philosopher and astronomer, and many others.

In fact, after several years working at Marāghah, this had prepared Quṭb ad-Dīn al-Shīrāzī quite well to write several important works on astronomy, such as *Nihāyat al-Idrāk fī Dirāyat al-Aflāk* (The Limit of Understanding of the Knowledge of the Heavens), *al-Tuḥfat al-Shāhiyya fi'l-Hay'ah* (The Royal Gift on Astronomy), *Kitāb faʿaltu fa-lā taʿlum fī'l-hay'ah* (A Book I Have Composed But Do Not Blame [Me for It], on Astronomy), and *Kitāb al-tabṣirah fī'l-hay'ah* (The Commentary on Astronomy). For the purpose of this article, we chose *Nihāyat al-Idrāk*, particularly the 6th chapter of the second treatise of the *Nihāyat*, which is on the motion and orbs of the sun. The translation of this chapter is based on the edited version of several manuscript copies of *Nihāyat al-Idrāk*, which we have edited for our PhD thesis, "An Edition, Translation, and Commentary of Chapters 6th, 7th, and 8th of Quṭb al-Dīn al-Shīrāzī's *Nihāyat al-Idrāk fī Dirāyat al-Aflāk*."

*Nihāyat al-Idrāk* is a very comprehensive work that covers not only mathematical astronomy but other areas of discipline that are quantitative and related to astronomy as well, such as geography, geodesy, meteorology, and mechanics. The whole text is divided into four treatises: "On what needs to be presented by way of introduction," "On the configuration of the celestial bodies," "On the configuration of the earth," and "On finding the measurements of the distances and the bodies." For this article, we chose Chapter Six from the second treatise, which is "On the configuration of the celestial bodies." As a word of caution, Quṭb ad-Dīn al-Shīrāzī's style of writing can be quite complicated and confusing. And this is even further complicated due to the subject matter itself, which is the science of astronomy that is still based on the Ptolemaic paradigm, i.e., the scientific astronomical paradigm that has been set by Claudius Ptolemy (d. c. 170 CE), the most important figure in Greek-Alexandrian astronomy. It is a paradigm that is based on the geocentric model of the orbs instead of the heliocentric model. However, this does not prevent the Arabic-Muslim astronomers, such as Quṭb ad-Dīn al-Shīrāzī and many others, from furthering the science of astronomy and thus contributing towards the revolution of the sciences later in the 16th and 17th centuries. In fact, we would discover some of Quṭb ad-Dīn al-Shīrāzī's contribution, particularly from this chapter, for example, his calculation in improving Ptolemaic's value of the precision of the fixed stars and the calculation of the longitude of the apogee of the sun.

Indeed, Quṭb al-Dīn al-Shīrāzī's knowledge in astronomy is deep and extensive as evident from the translation and commentary here. He lived at the time when Islamic civilization is still at its peak, and the knowledge of astronomy has advanced very highly due to the work of prominent Muslim astronomers such as Naṣīr al-Dīn al-Ṭūsī and Mūayyad al-Dīn al-'Urḍī. Both of them are responsible for building the famous Ilkhanid observatory at Marāghah in terms of constructing the instruments and conducting observations.

Quṭb al-Dīn al-Shīrāzī had the opportunity of studying under the tutelage of Naṣīr al-Dīn al-Ṭūsī at Marāghah, of which he was an outstanding student and later he himself had the responsibility of doing observations at the observatory. Indeed, to have such an extensive knowledge of astronomy as evident from his many works in astronomy, he must have done many observations to verify the astronomical theories. Moreover, the writing and the format of his *Nihāyat* is almost like the format of the famous work of his teacher al-Ṭūsī, which is *Tadhkīrah fī 'ilm al-hay'ah* (Memoirs in Astronomy).

There are quite a few parts in *Nihāyat* which are similar to the ones in *Tadhkīrah*. The latter, however, is a recession of astronomical knowledge without much attention being given to mathematical proofs. In *Nihāyat*, however, al-Shīrāzī would mention the theory, and following it is its explanation, and sometimes with observational evidence and mathematical proofs. Moreover, there are times when al-Shīrāzī would criticize his own teacher such as the non-circularity of al-Ṭūsī's planetary orb, and the impossibility of a solid sphere of an epicycle to be contained inside al-Ṭūsī 's couple.

However, despite of the extensive knowledge of astronomy that can be found in *Nihāyat*, al-Shīrāzī did not give much attempt to create his own theory. He depended heavily on the theories of his predecessors, and then would just elaborate further on the explanation. He seems to be satisfied with the solutions to the Ptolemaic problems as given by al-Ṭūsī and al-'Urḍī that he did not attempt to propose any new theory. However, shortly after finishing writing his *Nihāyat*, al-Shīrāzī wrote his other monumental work, *al-Tuḥfat al-shāhiyya fi'l-hay'ah* (The Royal Gift on Astronomy). Here in this work al-Shīrāzī proposed a new model for the moon and he applied both al-Ṭūsī's couple and al-'Urḍī's lemma to solve the irregular orb of Mercury's motion (Saliba, 1994).

**2. Translation.**

[1] When the situation of the sun was considered, the center of its body was found to adhere always to the ecliptic equator, deviating neither to the north nor to the south, since it was found that its altitude, when it is taken [at a certain moment when it is going] from

less to more but has not reached its maximum and [when it is taken at a certain moment when it is going] from more to less but has not reached its minimum, is equal to the complement of the latitude of the country; this indicates that the sun is at the vernal point in the first case and at the autumnal point in the second case (Ragep, 1993). Further, it was found that the altitude on the preceding day in the first case is less than the complement of the latitude of the country by an amount of the declination of the last degree of Pisces, and that in the second case it is greater than it by an amount of the declination of the last degree of Virgo. Moreover, it was found that the altitude on the next day in the first case is greater than [the complement of the latitude of the country] by [an amount of] the declination of the first degree of Aries, and that in the second case it is less than it by [an amount of] the declination of the first degree of Libra. This proves what we intended.

[2] Also, it was found that the sun's motion varies [in speed] in [different] parts of the ecliptic equator, in that it is slower in one half [of its orb] and faster in the other half, for the time between its arrival at the vernal equinox and its arrival at the autumnal descent was found to be greater than the time of [its traversing] the other half; likewise, the time between its arrival at the vernal equinox and its arrival at the summer solstice is greater than the time of [its traversing] the next quadrant. Also, it was found that in some eclipses, the sun's body during the middle of the period of slower motion was somewhat smaller than during the middle of the period of faster motion, since there existed a clear period of lingering of the eclipse, as has been perceived by Muḥammad bin Iṣḥāq al-Sharakhsī, during the middle of the period of slower motion, and there is a ring of remaining luminosity of the sun which encircles the moon, as has been observed by Abū al-'Abbās al-Īrānshāhrī, during the middle of the period of faster motion, although the distance of the moon [from the earth] is the same at both times. The moderns have proved from this that the sun is further away from the center of the world during the period of slower motion and closer to it during the period of faster motion. The ancients did not find that, as will be explained later in its place.

[3] Nevertheless, they concluded this, since the period of slower motion is longer than the period of faster motion; this shows what we intended to say, as you know. Moreover, the moderns found that the midpoints of its slower and faster motions, which are the apogee and the perigee [respectively]—and indeed, every position, whatever its circumstance, such as a specific speed or equation and such things—have a movement through the parts of the ecliptic equator approximately equal to the movement of the fixed stars due to the second motion. [This was found] by observing the amount of the sun's motion in a specific position of the ecliptic orb after leaving the vernal point and before it slows down to its extreme slowness, until it passes the extreme and arrives at the same condition as the first one; then it is known that the apogee is on the mid-point of the arc, which is between the two conditions, and that the perigee is opposite to it. Then the position of [the apogee] is observed after a period, and it is found that it has moved from the first position; then the arc that is between the two positions on the ecliptic orb is divided by the time between the two

observations. The result is that the [apogee's] motion is one degree in 66 Persian years; Ptolemy did not find that. This requires us to say that the sun does not have any other anomaly due to accession and recession and that the sun in its motion now is not faster than what it was at the time of Ptolemy, as some of them are believed to have established.

[4] [The above requires that there be established for the sun] [a] an eccentric, whose equator is in the plane of the ecliptic equator with the sun in its thickness, like a sphere immersed in water, whose depth is equal to the [sun's] diameter. The eccentric would move, and it would move the sun in the sequence of the zodiac in approximately 24 hours, 59 minutes, and 8 seconds. This is known by dividing one revolution, which is 360 degrees, by the single known period of return from the sun's arrival at the vernal point to its arrival back at the [same] point, which is approximately 365 days and a quarter day; the result is the motion for one day; it is called the motion of the sun's center or the uniform motion, but not the mean motion, as some have said, as will be mentioned later.

[5] or [b] an epicycle and deferent, whose equators are likewise [in the plane of the ecliptic equator]. The sun would be on the epicycle, and the latter would move it in its upper half counter-sequentially at the rate of the motion of the sun's center. The deferent moves the center of the epicycle sequentially, likewise at the rate of that motion [i.e., the motion of the sun's center], so that the two revolutions will be completed together. The center of the sun will undergo the same motion as the one produced by the eccentric, as mentioned before. This motion is slower in the apogeal half [of the deferent] and faster in the perigeal half. Ptolemy chose the first model without any necessity to do so, since it is simpler. For the eccentric model, there must be established a concentric orb in whose thickness the eccentric orb occurs, and which exceeds the eccentric by its two complementary bodies. It is called the parecliptic orb since its center, equator, and two poles correspond to those of [the ecliptic orb], or because on its circumference there is the circle, which is called 'representing', namely the ecliptic equator, according to what we have mentioned on what has also been said. According to the moderns, [this parecliptic orb] moves with the motion of the fixed stars, and it moves the apogee and the perigee.

[6] For the epicyclic model, the eighth orb suffices for the movement of the apogee and perigee, since it moves everything below it. For the eccentric model, it would also be sufficient, but since the existence of the parecliptic orb is necessary, it would not be proper to leave it idle. Thus, the motion of the fixed stars has been attributed to it. The statement that, when we know the number of movable objects due to the motions, then according to the opinion of Ptolemy, one does not need to establish the parecliptic orb, since according to him the apogee is fixed, is false. For the matter is just opposite, since if the sun is not posited to have a parecliptic orb, then it necessarily follows that the apogee is moving; if not, then it would follow that there would be a tearing.

[7] If [the sun] has a parecliptic orb, then it would be possible that [this orb] does not move. Then if someone asks how it is possible that the parecliptic orb does not move with the motion of

the eighth orb despite the fact that you say that the eighth orb is the mover for everything that is below it, then I would say: because it is possible to assume [that it is] such that [the eighth orb] moves all the parecliptic orbs, insofar as there is a soul connected with the eighth orb as well as the parecliptic orbs, and that it was inside it of what I know, and [one may also assume that it is such that the eighth orb] does not move any of [the parecliptic orbs], insofar as there is no soul connected to any [of the orbs]. According to this, when slow motion exists, it moves by itself, and what does not have that motion does not move with that motion, neither by itself nor accidentally. Consider this for the encloser and the enclosed: if both move around a center and around the same axis, then the enclosed, while it moves with its own proper motion, may also move with the motion of the encloser, even though it does not move by that motion. This is what we have to indicate, as required.

[8] Since the sun is always in the plane of the eccentric or epicyclic equator, which is itself in the plane of the parecliptic, it does not have any latitude. We have set forth the illustration of the sun's two orbs according to the eccentric model, as was Ptolemy's preference, and most of the moderns have chosen it. Some of them chose the epicyclic model, and among them is the master of the main disciples. It has been presented before that if the matter of preference were established [either for the eccentric model or for the epicycle model], then preference would be given to the first model; it is even a necessary outcome.

[9] The sun has a single anomaly equal to the amount by which the sun's observed motion, which is an arc on the ecliptic orb between the beginning of Aries and the end of the line that is extended from the center of the world to the center of the sun's body and from there to the ecliptic orb, differs from its mean motion, which is an arc on the ecliptic orb between the beginning of Aries and the end of the line that is extended from the center of eccentric to the center of the sun's body and from there to the ecliptic orb. [This anomaly] is an angle called the angle of the equation, and it is formed at the center of the sun between the two lines mentioned [above]. It attains its greatest value at the two mean distances based on the motion [of the sun on the eccentric], and it disappears at the two other distances. The maximum value [of the anomaly] depends on the amount of eccentricity that is required between the two centers. According to Ptolemy, it is 2; 30, and according to the observations of modern astronomers, it is about 2; 05, with the radius of the eccentric orb being 60 parts. If the radius of the parecliptic orb is taken as 60 parts, then it is said that [the eccentricity] is 2; 01. The first amount is used for obtaining the equation, and the second one is used for obtaining the distance of the sun from the earth. We shall have to look into this since the first amount is used for obtaining the sun's distance from the earth according to what will be mentioned in [the chapters] on distances and bodies.

[10] The position of the apogee, according to Ptolemy, is 5½˚ after [the beginning of] Gemini. According to the moderns, the value varies, as they have stated in their *zīj*es, depending on the date; according to the latest observation, the position of the apogee amounts to 27˚ 6' of Gemini at the end of the year 650 of Yazdegard. The well-known mean distance, which is

generally accepted, is where the two lines that are extended from the two centers are equal. They are the two points on the circumference of the eccentric equator, which are the end points of the perpendicular line, which is extended from the mid-point of the two centers, perpendicular to the line connecting those centers. This is the mean distance based on distance [to the earth], since the distance between the center of the world and [to the point on the circumference] is half [of the sum] of the distances between the center of the world and the nearest and furthest distances. Thus, it is said that it is derived from the mean number because that is also half of the sum of its two extremes, like 5 is half of [the sum of] 4 and 6, and it is also likewise [half of the sum of] 3 and 7, and [half of the sum of] 2 and 8, and [half of the sum of] 1 and 9.

[11] What has been presented before is the mean distance based on motion since [the sun's motion] is at the mean between the faster motion and the slower motion. Al-Mas'ūdī said that if the planet is at the mean [distance between] the apogee and perigee such that its distance from the apogee is equal to its distance from the perigee, then it is said that [the planet] is at the mean distance. If he meant [that one measures] one quarter of a circle from the apogee with respect to the center of the world, then [the planet is] at the mean distance as based on motion. However, if he meant that one measures one quarter of a circle from the apogee with respect to the center of eccentricity, then it is a saying that nobody has ever propounded.

[12] The above having been determined, it should be known that the following concepts are well-known: The solar apogee is the arc measured sequentially on the parecliptic between the first of Aries and the apogee point. The solar center, which is also called its proper anomaly, is the arc measured sequentially on the eccentric between the apogee and the center of the sun. The mean sun is the sum of these two arcs, and it is the simple compounded motion, the explanation of which was promised. All means of the planets are likewise, and the way to add [the angles], according to what has been mentioned, is that one imagines an angle between two lines extending from the center of the world to two limiting points of the motion of the apogee, and another angle between the two lines extending from the center of the eccentric to two limiting points of the motion of the center [of the planet] in that time; then both angles are added, considering that a perpendicular [angle] is 90˚; then what has results is the mean [motion].

[13] The true position is an arc on the parecliptic between the first of Aries and the endpoint of the line extended from the center of the world to the sun's body. This will be less than the mean by the amount of the anomaly, which is called the equation, if the sun is descending; this is due to the fact that the end of the line, which is extended from the center of the world, is closer to the apogee than [the end of the line], which is extended from the center of eccentric. [However, the true position] is greater than [the mean] if the sun is ascending due to the opposite of what we just said. If someone said that the anomaly must be subtracted from the mean [position] if the sun is descending and added to [the mean position] if the sun is ascending in order to produce the true position, then how to use the anomaly if the first of Aries is at the mid-point of the anomaly, which is additive [to the

mean position]? Let's say that it is 2˚, while the mean position [is measured] from the midpoint of the anomaly to the endpoint of the line extended from the center of eccentric to the parecliptic orb. The answer to this is that this mean position is then one revolution minus 1˚; then the anomaly, which is 2˚, is added to this mean position, and the sum is one revolution plus 1˚. Then one revolution is deducted [from the sum], and the remaining [result] is the true position.

[14] One may also say that the center of the mean sun is the arc on the eccentric orb between the sun's apogee and the center of the sun's body, and its equation is the mentioned angle. The corrected center [of the sun] is the difference between the [sun's] equation and the center of the mean [sun], or the sum of both. It is measured by the angle that is formed at the center of the world between the two lines that are extended from it, one of them to the sun's apogee and the other to its center. This determines the [sun's] position on the ecliptic. From the sum of the distance of the apogee from Aries and [the distance of] the corrected center [of the sun] will result in [the distance of] the position of the sun on the ecliptic from the head of Aries; this is its true position. This is a well-known [fact].

[15] According to the investigators, one of them being Ptolemy, the mean sun is an arc on the circle of the ecliptic between the first of Aries and the end of the line, which extends from the ecliptic center [i.e., the center of the world] to the [ecliptic's] circumference and which is parallel to the line that connects the centers of the deferent and the sun, or coincides with it. [This arc] is equal to the arc on the eccentric between the straight line that extends from the eccentric center to its circumference, and which is parallel to the straight line that extends from the ecliptic center to the first of Aries and the center of the sun. The sun's mean anomaly is an arc on the ecliptic between the line that passes through the centers of the ecliptic and the deferent and is extended further to the ecliptic orb and the line that extends from the ecliptic center and is parallel with the line that connects the centers of the sun and the deferent. [This arc] is the same as the remaining arc of the mean [longitude] after the apogee's [longitude] has been deducted from it. The [sun's] equation is an arc on the ecliptic orb between the two mentioned lines that extend from the ecliptic center to the [ecliptic's] circumference, one of them passing through the center of the sun and the other being parallel to the line that connects the centers of the deferent and the sun; its amount equals the angle that is bounded by these two lines at the center of the ecliptic.

[16] Let us draw a figure to clarify the meaning. We say that if we assume K to be the first of Aries and B is the sun, then [the arc for] the mean [sun] is KAR, and [this arc] is equal to [the arc] LAB on the eccentric. The [sun's] mean anomaly is AR, and the [sun's] equation is the arc ṬR, and the angle of the equation is ṬHR. It is not hidden that the point R cuts from the ecliptic arcs that are equal to [the arcs] that the sun cuts from the eccentric. Thus, the sun, in its mean [position] cuts from the ecliptic equal arcs at equal times. Know this, for you will need it in [the case of] the moon. Ptolemy worked this out in order to derive everything from one circle, not to do the correct work, according to the beliefs of some moderns. It is repulsive to others in that they made ṬḤ, the arc of the

equation, which is in all places less than the true equation; they said that it is equal to the angle ’ABH, and it is not known to them that this angle does not measure an arc on the ecliptic, insofar as it is not viewed from the center [of the ecliptic]. Since that is the case, the work is not correct, and what is not correct is rejected; therefore, it leads to the same thing.

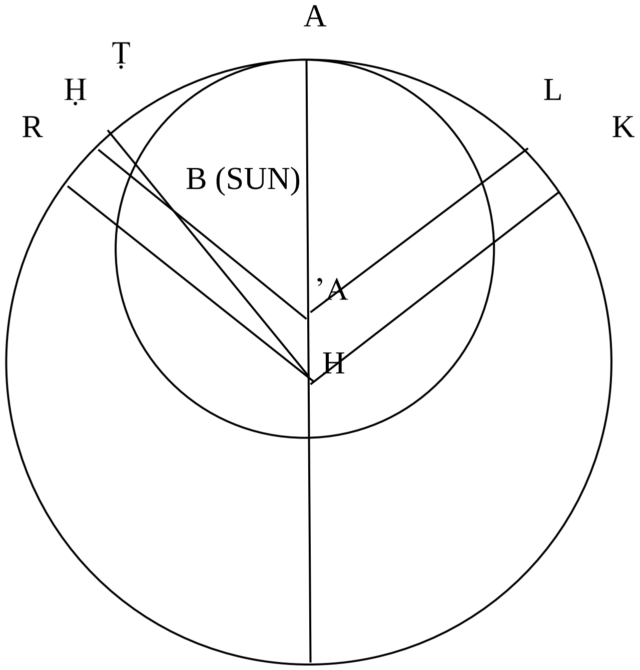


**Figure 1**

[17] Thus, the true position [of the sun] in the figure is KAT, irrespective of whether we say that the equation is ṬḤ or ṬR. Then the repulsiveness does not occur; the one for whom this is repulsive is the one who does not know that the angle ’ABH is not [measured] by the amount of the arc ṬḤ, but since they know that it is equal to the true angle of equation, which is ṬHR, since it is an alternate angle [with ’ABH], and that ṬḤ is judged as ṬR in the increase and the decrease [of the equation] in all positions [on the ecliptic orb], they considered ’ABH as the angle of the equation and ṬḤ as the arc of the equation.

[18] Know that the mean position, the equation, and the [solar] center in the model of the deferent and epicycle differ from the mean position, the equation, and the [solar] center in the model of the eccentric; this will be explained in the next chapters. Thus, the case of the sun has been ordered by two orbs and two motions, if we say that the sun has only one anomaly, as is well-known, and this is adopted by the general public and the geometers who confine themselves to two circles; one of the circles is the eccentric equator with the condition that the center of the sun is on it, and the other circle is the parecliptic equator with the condition that it touches the eccentric equator. This is the figure of the solid orbs of the sun based on the eccentric model according to what can be drawn in a plane; the black circles are those to which the geometers confine themselves. This is the end of what we intended to bring up in this chapter.

**3. Commentary.**

[1] The sun is on the celestial equator when its altitude is equal to the complementary latitude of the country considered. The complement latitude of the country is equal to 90˚ minus the latitude of the country, i.e., it is the maximum altitude of the celestial equator at that position on earth.

[2] The longitude of the sun's vernal descent to its summer descent is from 0˚ to 60˚, i.e., from Aries to Cancer. The next quadrant is the quadrant from autumnal descent to winter descent, i.e., from 180˚ to 270˚, which is from Libra to Capricorn. The sun takes longer to pass through in the former compared to the latter. While the time for the sun to travel from summer descent to autumnal descent is equal to the former, the time for it to travel from winter descent to vernal descent is equal to the latter.

Muḥammad bin Iṣḥāq al-Sharakhsī was a mathematician and astronomer who lived at the time of Abū Rayḥān al-Bīrūnī (d. ca. 1050) (Sezgin, 1974). The latter also cited him in his *al-Qānūn al-Mas'ūdī.*

[3] Ptolemy was aware that the fixed stars have their own motion, which is called precession, and if the sun makes one complete revolution from one fixed star and back to the same star, the time period is called one sidereal year. Ptolemy did not use this time period since he assumed a constant rate of precession, which is very small, about 1˚ per century, and thus it is convenient to assume that the apogee is fixed among the fixed stars. The modern value for precession is about 1˚ per 71 years (Pedersen, 1974).

[4] The amount here is only approximate. In the Almagest, the sun's mean motion is 0˚; 59, 8, 17, 13, 12, 31 per day, with a solar year of $365^d\ 5^h\ 55^m\ 12^s$. Al-Bīrūnī has better values; in his book *The Book of Instruction on the Elements of the Art of Astrology*, the sun's mean motion is 0˚; 59, 8, 23 per day with a solar year of $365^d\ 5^h\ 47^m$. His solar year is closer to the modern value of $365^d\ 5^h\ 48^m\ 46^s$ (Pedersen, 1974).

[5] Since the sun has a single anomaly, it can have either an epicyclic model or an eccentric model; both are interchangeable. Ptolemy preferred the eccentric model, and so did most of the Arabic astronomers.

[9] The angle of the equation of the sun's anomaly attains its maximum value at the mean distances on the eccentric orb between the apogee and perigee, and it disappears at these two points. The maximum value depends on the eccentricity, which is the distance between the center of the world and the eccentric center.

[10] Ptolemy's longitude of the apogee is 65˚; 30, which he has taken from Hipparchus's observation about 300 years ago. This is erroneous, since due to the precession of the fixed stars, the apogee's longitude should have been about 70˚ during the time of Ptolemy. Hence, Quṭb al-Dīn al-Shīrāzī was right that at his time, according to his latest observation, the apogee's longitude is 87˚; 6'; and this is due to the fact that there are about 1300 years between Ptolemy and al-Shīrāzī (Pedersen, 1974).

Hipparchus was another famous Greek astronomer who lived around the 2nd century B.C. and whose observations were heavily relied on by Ptolemy in constructing his astronomical theories.

[11] The midpoint of the anomaly is the first of Aries. Hence, the amount of the mean position is equal to 360˚ - 1˚ = 359˚. To find the true position, we added the amount of this mean position, 359˚, with the amount of the anomaly, 2˚, of which from the result, which is 361˚, we deducted 360˚, to lead us to the amount of the true position of 1˚, i.e., 359˚ + 2˚ - 360˚ = 1˚.

[12] Here, the term of the mean sun is meant to be the mean anomaly, $a_m(t)$, i.e., an arc on the ecliptic orb from the apogee to the mean sun. The corrected center of the sun is the true anomaly, a(t), which is an arc on the ecliptic orb from the apogee to the true sun, and both anomalies are related to the sun's equation by a formula of $a(t) = a_m(t) \pm q$, of which q is the sun's equation. The true longitude of the sun is the sum of the longitude of the apogee with the true anomaly, $\lambda t = \lambda_A + a(t)$ (Pedersen, 1974).

[13] Likewise, the mean anomaly, $a_m(t)$, is related to the mean longitude of the sun, $\lambda_m$, and the apogee's longitude as $a_m(t) = \lambda_m - \lambda_A$.

[14] In Figure 1, A is the sun's apogee. ṬR is the arc for the sun's equation, and ṬHR is the angle of the equation. However, some of the Arabic scholars erroneously assumed that ṬḤ is the arc for the equation since it is assumed that it is equal to the angle of the equation, ʼABH.

[15] It is clear from Figure 1 that the arc ṬḤ does not measure the angle of the equation ʼABH or ṬHR since the lines that are extended to the two points Ṭ and Ḥ are not extended from the center of the ecliptic, although the fact is that the arc ṬḤ occurred on the ecliptic.

## 4. Conclusion.

Although Quṭb al-Dīn al-Shīrāzī, just like many other Arabic-Muslim astronomers, worked and contributed under the geocentric model of the universe, this does not prevent him and many other Arabic-Muslim astronomers from contributing and improving the values and technique of observations that had been set by Ptolemy about a thousand years ago. As mentioned in this Chapter Six of *Nihāyat al-Idrāk*, Quṭb al-Dīn al-Shīrāzī had improved values for the precision of the fixed stars and the longitude of the apogee of the sun. In fact, from his other writings, Quṭb al-Dīn al-Shīrāzī was also known as one of the main Arabic-Muslim astronomers who revolutionized the Ptolemaic model, especially in the model of the motion of Mercury. Other astronomers who had also contributed with their own planetary model include Naṣīr al-Dīn al-Ṭūṣī, Mūayyad al-Dīn al-ʻUrḍī, and Ibn al-Shāṭir (d. 1375 CE), a 14th-century Damascene astronomer, whom it is said had been highly influential in the Copernican revolution. Thus, it is important to appreciate the contribution of the Arabic-Muslim astronomers, which eventually led to a complete reform of Ptolemaic astronomy, i.e., from a geocentric model to a heliocentric model.

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