Nonlinear Consensus for Multi-Agent Systems using Positive Intractions of Doubly Stochastic Quadratic Operators

Rawad Abdulghafor, Sherzod Turaev, Mohd Izzuddin

Kulliyyah of Information and Communication Technology,
International Islamic University Malaysia, 53100, Kuala Lumpur, Malaysia.
raaac2004@yahoo.com, {sherzod, izzuddin}@iium.edu.my

Abstract— This technical note addresses a new nonlinear protocol class based on doubly stochastic quadratic operators (DSQO) for a consensus problem in multi-agent systems (MAS). In this technical note, a faster consensus is achieved for MAS by the rules of DSQO with the positive interactions among agents.

Keywords— consensus problem, multi-agent systems, positive interactions doubly stochastic quadratic operators.

I. Introduction

In recent years, multi-agent systems MAS consensus problem has been an attractive research topic due to the prospective applications in unmanned aerial vehicles (UAV), sensor networks fields, distributed computing in computer science, medicine, environmental monitoring and military reconnaissance [1–8]. In general, the motivation of interest in distributed system is driven by the coordination and manage of various agents in the great information on access to large-scale networks to reach an accepted decision (value) or consensus on a common convergence. Numerous results were found in this area. The consensus problem has a long history [9]. For instance, DeGroot model was considered in [10]. A distributed computing over networks was presented in [11]. Problems with asynchronous parallel computing were studied in [12] and [13]. Vicsek [14] studied a specific situation model in which all players move at the same constant speed and maintain their positions in the government closest neighbor. Behavior cohesion and speed flocking were established in both cases, and observed convergence test was provided [15]. Fax and Murray [16] proposed formations Multivehicle control study of the fixity of Nyquist with a standard. A theoretical framework to solve the consensus problem was introduced in Saber and Murray [17]. Cao et al. [18] highlighted a graphical approach of a linear model to a consensus in a dynamically changing environment. Lin and Ren [19] studied the problem of stress MAS consensus to dynamically change the asymmetric networks with delays in communication. Shi and Johansson [20] discussed a linear time consensus problem with stochastic matrix with positive diagonal. Hu et al. [21] studied the dynamics of a linear general consensus controlled by the required pattern triggered event functionality with any distribution.

However, the above studies were built upon the assumption that the dynamic agents of consensus protocols are linear. This estimation cannot always be satisfied due to the fact that engineering of the physical system is a particular type of problem consensus [22] [23].

Numerous studies have tried to take into account the non-linear convergence protocols for the problem of consensus in the MAS. The nonlinear system poses challenges to investigate the problem of consensus of a static graphic for nodes [24]. Early research on the theory of nonlinear stability control was considered in [25]. Murray [26] introduced a linear and nonlinear protocols for a consensus agreement in distributed systems and proposed cooperation. The analysis of nonlinear consensus protocols is considered in the case of applications such as active agents are physical models that consider entry restrictions. Nonlinear discrete-time structure was introduced in [27]. Meanwhile, Lin et al. [28] showed that the consensus is executed nonlinear subsystems only when agents have sufficient dynamic interaction relationship.

The new technique of various nonlinear consensus have established by [29–35] for the consensus problem of cooperative agents in network.

Nevertheless, the disadvantage of nonlinear models is that they often are more complex and configured with restricted conditions. The current concern is to explore possible nonlinear models with faster convergence to a still relatively low and more resilient conditions optimal complexity consensus.

In this arterial, a model of the nonlinear quadratic operator doubly stochastic (DSQO) with positive interactions has been studied to increase control problem consensus on lower more flexible terms and complexity for specific subclass of DSQO.

II. Methodology

The class of DSQO is tracked back to [36]. It called bistochastic quadratic operators, where the theorem of a necessary and sufficient conditions were peeved for bistochastic quadratic operators. These theorems also was obtained in [37] [38]. The concept of DSQO are related to the majorization notation in [39]. DSQO have applied for the problem in population genetics [40]. The matrices by the notations of majorization have called the welfare operator. The welfare operator was applied for the problem in economic [39]. The nonlinear discrete dynamic systems of DSQO are defined in [41], [42].

Definition 1: A (m-1)- dimensional simplex is a set

$$S^{m-1} = \{x = (x_1, x_2, \dots, x_m) \in R^m : x_i \geq 0, 1 \leq i \leq m, \sum_{i=1}^{m} x_i = 1\} \quad (1)$$

The DSQOs are related to population evolution. It considers a population consisting of m species. Let $x^0 = (x_1^0, x_2^0, \dots, x_m^0)$ be the probability distribution of species in the initial generations, and $P_{ij,k}$ be the probability that individuals in the i_{th} and j_{th} species interbreed to produce an individual k. This probability is denoted (the heredity coefficient) via $P_{ij,k}$ and $\sum_{k=1}^{m} p_{ij,k} = 1$ for all i, j, that is, x_i and x_j are the fractions of species i and j in the population. In this case, parent pairs i and j arise for a fixed state $x = (x_1, x_2, \dots, x_m) \in R^m$ with probability $x_i x_j$ [43].

Definition 2: A DSQO V: $S^{m-1} \rightarrow S^{m-1}$ is defined as ([41])

$$(Vx)_k = \sum_{i,j,k=1}^{m} p_{ij,k}\, x_i x_j, \quad p_{ij,k} \geq 0, \quad \text{for all } \; i, j, k \in \{1, \dots, m\} \qquad (2)$$

where coefficients $p_{ij,k}$ satisfy the following conditions ([44], [45]):

$$p_{ij,k} = p_{ij,k} > 0, \sum_{k=1}^{m} p_{ij,k} = 1, \qquad (3)$$

More strictly, $V: S^{m-1} \rightarrow S^{m-1}$ is a DSQO:

$$V(x_i^{(t+1)}) = \begin{pmatrix} \sum_{i,j=1}^{m} x_i^t p_{ij,1}\, x_j^t, \sum_{i,j=1}^{m} x_i^t p_{ij,2}\, x_j^t, \dots, \\ \sum_{i,j=1}^{m} x_i^t p_{ij,m}\, x_j^t \end{pmatrix} \qquad (4)$$

where the x_i^t is a row vectors of the agents' status and the x_j^t is a column vectors of the statuses agents, while the $p_{ij,k}$ is the transition matrices where k means that for each agent there is a separate transition matrix.

The new notation is included in the proposed work for DSQO to achieve the consensus is that the weighted values of the distribution matrices are positive. The matrices of DSQO must belong to the set U_p where the elements of the matrices are positive.

So in a finite dimensional (nD) will have n matrices each of size $n \times n$ and they satisfy the following conditions (set U_p):

$$U_p = \{A = (a_{ij}) : a_{ij} = a_{ji} > 0, \sum_{ij \in \propto} | \propto |, \sum_{ij \in I} a_{ij} = m\} \quad (5)$$

where $\propto \subset I = \{1, 2, \dots, m\}$.

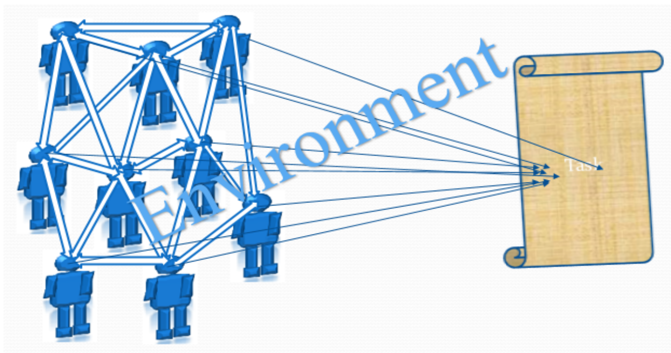

Fig. 1 The Consensus in Multi-Agent Systems.

As presented in the Fig. 1 the communication in the networks of the agents is the interactions which is the transition matrix under rules setU_p, while the initial statuses are belong to Equation 1.

III. Result and Simulation

In this section, we provide some examples of simulation for nonlinear protocols proposed lighting efficiency DSQO when all elements of the transition matrix are positive (meaning that each agent has a positive interaction with all agents).

The simulation of the consensus of DSQO confirms that MAS consists of 100, 200 and, 300 converges to average consensus $(\frac{1}{m})$ as in the Figures 2, 3 and 4. Moreover, it shown that the consensus of DSQO has achieved fast to the average value.

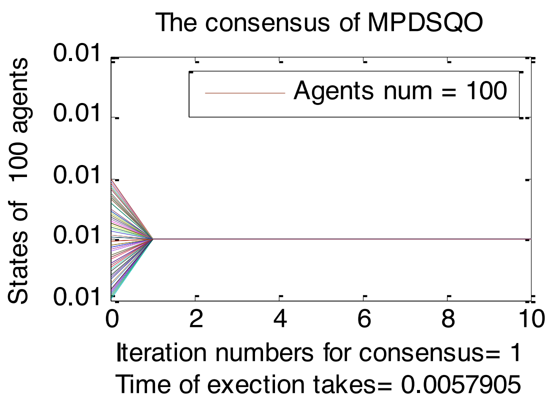

Fig. 2 The Consensus of 100 agents by DSQO.

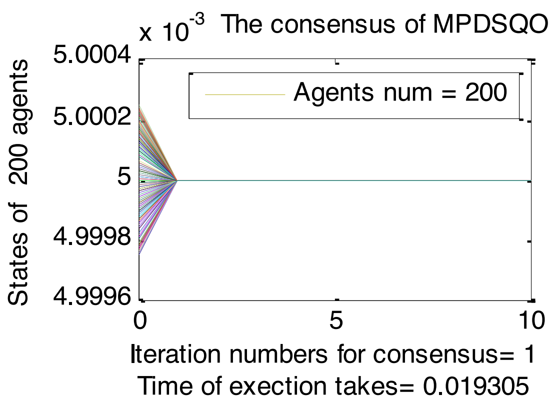

Fig. 3 The Consensus of 200 agents by DSQO.

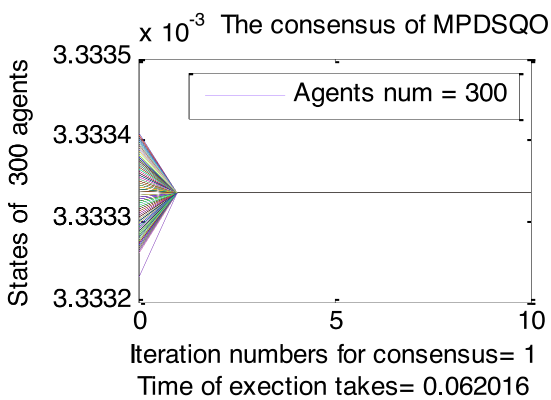

Fig. 4 The Consensus of 300 agents by DSQO.

IV. Conclusions

In this article, a nonlinear protocol for the consensus problem in MAS, which generalizes linear and nonlinear protocols, had been established. The interaction among agents that were proposed here, found to be positive. We disclosed that the MAS reaches a consensus in faster time from any initial status and from any random weighted values of the interaction among agents under DSQO protocol if each member of the agent group has positive interactions with the others.

References

[1] W. Yu, G. Chen, and M. Cao, "Consensus in directed networks of agents with nonlinear dynamics," *Autom. Control. IEEE Trans.*, vol. 56, no. 6, pp. 1436–1441, 2011.

[2] V. Schwarz and G. Matz, "Nonlinear average consensus based on weight morphing.," in *ICASSP*, 2012, pp. 3129–3132.

[3] Z. Li, W. Ren, X. Liu, and M. Fu, "Consensus of multi-agent systems with general linear and Lipschitz nonlinear dynamics using distributed adaptive protocols," *Autom. Control. IEEE Trans.*, vol. 58, no. 7, pp. 1786–1791, 2013.

[4] D. Ding, Z. Wang, B. Shen, and G. Wei, "Event-triggered consensus control for discrete-time stochastic multi-agent systems: The input-to-state stability in probability," *Automatica*, vol. 62, pp. 284–291, 2015.

[5] G. Wen, Y. Yu, Z. Peng, and A. Rahmani, "Distributed finite-time consensus tracking for nonlinear multi-agent systems with a time-varying reference state," *Int. J. Syst. Sci.*, vol. 47, no. 8, pp. 1856–1867, 2016.

[6] X. Dong, B. Yu, Z. Shi, and Y. Zhong, "Time-varying formation control for unmanned aerial vehicles: theories and applications," *Control Syst. Technol. IEEE Trans.*, vol. 23, no. 1, pp. 340–348, 2015.

[7] N. Amelina, A. Fradkov, Y. Jiang, and D. J. Vergados, "Approximate consensus in stochastic networks with application to load balancing," *Inf. Theory, IEEE Trans.*, vol. 61, no. 4, pp. 1739–1752, 2015.

[8] L. Zheng, Y. Yao, M. Deng, and S. S. T. Yau, "Decentralized detection in ad hoc sensor networks with low data rate inter sensor communication," *Inf. Theory, IEEE Trans.*, vol. 58, no. 5, pp. 3215–3224, 2012.

[9] M. H. DeGroot, "Reaching a Consensus," *J. Am. Stat. Assoc.*, vol. 69, no. 345, pp. 118–121, 1974.

[10] R. L. Berger, "A necessary and sufficient condition for reaching a consensus using DeGroot's method," *J. Am. Stat. Assoc. Taylor \& Fr. Group.*, vol. 76, no. 374, pp. 415–418, 1981.

[11] V. B. V. Borkar and P. V. P. Varaiya, "Adaptive control of Markov chains, I: Finite parameter set," *1979 18th IEEE Conf. Decis. Control Incl. Symp. Adapt. Process.*, vol. 18, no. 6, pp. 2–6, 1979.

[12] J. N. Tsitsiklis, "Problems in Decentralized Decision making and Computation," *Tech.rep.*, p. DTIC Document, 1984.

[13] J. N. Tsitsiklis, D. P. Bertsekas, and M. Athans, "Distributed Asynchronous Deterministic and Stochastic Gradient Optimization Algorithms," *1986 Am. Control Conf.*, no. September, 1986.

[14] T. Vicsek, "Novel type of phase transition in a system of self-driven particles}," *Phys. Rev. Lett.*, vol. 75, no. 4, pp. 729–732, 1995.

[15] J. and others Jadbabaie, Ali and Lin, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *Autom. Control. IEEE Trans. on, IEEE.*, vol. 48, no. 6, pp. 988–1001, 2003.

[16] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *Autom. Control. IEEE Trans.*, vol.

49, no. 9, pp. 1465–1476, 2004.

[17] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *Autom. Control. IEEE Trans.*, vol. 49, no. 9, pp. 1520–1533, 2004.

[18] M. Cao, A. S. Morse, and B. D. O. Anderson, "Reaching a consensus in a dynamically changing environment: A graphical approach," *SIAM J. Control Optim.*, vol. 47, no. 2, pp. 575–600, 2008.

[19] P. Lin and W. Ren, "Constrained consensus in unbalanced networks with communication delays," *Autom. Control. IEEE Trans.*, vol. 59, no. 3, pp. 775–781, 2014.

[20] J. M. Hendrickx, G. Shi, and K. H. Johansson, "Finite-time consensus using stochastic matrices with positive diagonals," *Autom. Control. IEEE Trans.*, vol. 60, no. 4, pp. 1070–1073, 2015.

[21] W. Hu, L. Liu, and G. Feng, "Consensus of linear multi-agent systems by distributed event-triggered strategy," *Cybern. IEEE Trans.*, vol. 46, no. 1, pp. 148–157, 2016.

[22] G. Cui, S. Xu, F. L. Lewis, B. Zhang, and Q. Ma, "Distributed consensus tracking for non-linear multi-agent systems with input saturation: a command filtered backstepping approach," *IET Control Theory Appl.*, 2016.

[23] L. Yu-Mei and G. Xin-Ping, "Nonlinear consensus protocols for multi-agent systems based on centre manifold reduction," *Chinese Phys. B*, vol. 18, no. 8, p. 3355, 2009.

[24] Z. Lin, B. Francis, and M. Maggiore, "State agreement for continuous-time coupled nonlinear systems," *SIAM J. Control Optim.*, vol. 46, no. 1, pp. 288–307, 2007.

[25] P. Kokotović and M. Arcak, "Constructive nonlinear control: a historical perspective," *Automatica*, vol. 37, no. 5, pp. 637–662, 2001.

[26] R. O. S. R. M. Murray, "Consensus protocols for networks of dynamic agents," in *Proceedings of the 2003 American Controls Conference*, 2003.

[27] L. Moreau, "Stability of multiagent systems with time-dependent communication links," *IEEE Trans. Automat. Contr.*, vol. 50, no. 2, pp. 169–182, 2005.

[28] Z. Lin, B. Francis, and M. Maggiore, "State agreement for continuous-time coupled nonlinear systems," *SIAM J. Control Optim.*, vol. 46, no. 1, pp. 288–307, 2007.

[29] W. Yu, G. Chen, M. Cao, and J. Kurths, "Second-Order consensus for multiagent systems with directed topologies and nonlinear dynamics," *IEEE Trans. Syst. Man, Cybern. Part B Cybern.*, vol. 40, no. 3, pp. 881–891, 2010.

[30] A. N. Bishop and A. Doucet, "Distributed nonlinear consensus in the space of probability measures," *arXiv Prepr. arXiv1404.0145*, 2014.

[31] M. Zhu and S. Martínez, "Discrete-time dynamic average consensus," *Automatica*, vol. 46, no. 2, pp. 322–329, 2010.

[32] A. Ajorlou, A. Momeni, and A. G. Aghdam, "Sufficient conditions for the convergence of a class of nonlinear distributed consensus algorithms," *Automatica*, vol. 47, no. 3, pp. 625–629, 2011.

[33] S. Bolouki, "LINEAR CONSENSUS ALGORITHMS : STRUCTURAL PROPERTIES AND," 2014.

[34] D. Meng, Y. Jia, J. Du, and J. Zhang, "On iterative learning algorithms for the formation control of nonlinear multi-agent systems," *Automatica*, vol. 50, no. 1, pp. 291–295, 2014.

[35] L. Buoniu and I. C. Morrescu, "Consensus for black-box nonlinear agents using optimistic optimization," *Automatica*, vol. 50, no. 4, pp. 1201–1208, 2014.

[36] R. N. Ganikhodzhaev, "On the definition of bistochastic quadratic operators," *Russ. Math. Surv.*, vol. 48, no. 4, pp. 244–246, 1993.

[37] F. Shahidi, "On the extreme points of the set of bistochastic operators," *Math. Notes*, vol. 84, no. 3–4, pp. 442–448, 2008.

[38] F. A. Shahidi, "Doubly stochastic operators on a finite-dimensional simplex," *Sib. Math. J.*, vol. 50, no. 2, pp. 368–372, 2009.

[39] I. Olkin and A. W. Marshall, "Inequalities: Theory of majorization and its applications," *Acad. New York*, 1979.

[40] R. N. Ganikhodzhaev, "On the definition of bistochastic quadratic operators," *Russ. Math. Surv. Turpion Ltd.*, vol. 48, no. 4, pp. 244–246, 1993.

[41] R. Ganikhodzhaev and F. Shahidi, "Doubly stochastic quadratic operators and Birkhoff's problem," *Linear Algebra Appl.*, vol. 432, no. 1, pp. 24–35, 2010.

[42] R. N. Ganikhodzhaev, "Quadratic stochastic operators, Lyapunov functions, and tournaments," *Russ. Acad. Sci. Sb. Math.*, vol. 76, no. 2, p. 489, 1993.

[43] R. N. Ganikhodzhaev and U. A. Rozikov, "Quadratic Stochastic Operators: Results and Open Problems," *arXiv Prepr. arXiv0902.4207*, 2009.

[44] F. Shahidi, "Necessary and sufficient conditions for doubly stochasticity of infinite-dimensional quadratic operators," *Linear Algebra Appl.*, vol. 438, no. 1, pp. 96–110, 2013.

[45] R. Ganikhodzhaev and F. Shahidi, "Doubly stochastic quadratic operators and Birkhoff's problem," *Linear Algebra Appl.*, vol. 432, no. 1, pp. 24–35, 2010.