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Assessing the Psychometric Properties of an Instrument Measuring Instructional Strategies, Algebraic Thinking and Flexible Mathematical Thinking: A Pilot Study

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Abstract

This paper reports on the results of a pilot study conducted at the International Islamic University Malaysia (IIUM), Kuantan Campus to examine the clarity, reliability, and internal consistency of a survey instrument measuring instructional strategies, algebraic thinking, and flexible mathematical thinking among undergraduate students. The sample comprised 33 students of Mathematics who responded to a 47-item Likert scale on the three constructs. Data were analyzed using SPSS version 30 and Partial Least Squares Structural Equation Modeling (PLS-SEM) with SmartPLS version 4. The reliability analysis revealed strong internal consistency for each construct: instructor-based instruction (5 items, $\alpha = 0.78$), learner-focused instruction (9 items, $\alpha = 0.85$), generalized arithmetic (9 items, $\alpha = 0.90$), functions (9 items, $\alpha = 0.93$), modelling (5 items, $\alpha = 0.90$), and flexible mathematical thinking (10 items, $\alpha = 0.90$). Convergent validity was supported, with most factor loadings above 0.50 and Average Variance Extracted (AVE) values ranging from 0.52 to 0.72, demonstrating that the items adequately captured the intended constructs. Discriminant validity was confirmed through cross-loadings, the Fornell–Larcker criterion, and HTMT ratios below 0.85. Full collinearity VIF values were examined to assess potential common method bias, with all VIFs below the recommended threshold of 3.3 (ranging 1.00–2.47), indicating minimal risk of CMV. Three items were removed due to low loadings, further improving model validity. The results suggest that the instrument can be effectively used to assess instructional strategies, algebraic thinking and mathematical thinking among undergraduates, providing insights for curriculum design and pedagogical practices. Further studies are necessary to examine the predictive validity of this instrument, particularly within mathematics classrooms.

Keywords: *Instructional strategies, algebraic thinking, flexible mathematical thinking, Mathematics education, SmartPLS.*

INTRODUCTION

The influence of mathematics on the progress of human civilization is both profound and well-documented. Beyond its practical applications in fields such as economics, engineering, and the arts, mathematics facilitates the development of essential cognitive skills such as logical reasoning, critical thinking, and creativity (Laurens et al., 2018). Sir Wilfred Cockcroft, an eminent mathematics educator who led the Cockcroft Report, emphasized that mathematics should be taught in schools because it serves as a precise form of communication, allows information to be represented in multiple ways, and brings intellectual satisfaction through problem-solving (Committee of Inquiry into the Teaching of Mathematics in Schools, 1982). Accordingly, mathematics should be introduced early in schooling and sustained throughout all levels, as it trains the mind to think logically, to reason critically, and to solve problems systematically (Ahdhianto et al., 2020). The importance is highlighted by international bodies such as the National Council of Teachers of Mathematics (NCTM, 2000) and the OECD (2021), which emphasize its role in developing mathematical literacy, higher-order thinking, and flexible mathematical thinking. The ability to solve non-routine, ill-structured problems is increasingly recognized as an crucial skill required for lifelong learning, particularly within STEM fields (Bayat & Tarmizi, 2010; Parmjit et al., 2016; Schoenfeld, 1992).

Reflecting this global emphasis, Malaysia's education policy frameworks, outlined in the Malaysian Education Blueprint 2013–2025, highlight the importance of integrating analytical and problem-solving skills into the mathematics curriculum to prepare learners for the demands of the 21st century society and workforce (Malaysian Ministry of Education, 2013). According to reports by Bernama (2025), ongoing development of the future new Malaysia Future Education Blueprint (2026–2036) also aims to reform education quality, relevance, and STEM learning outcomes in line with global expectations. Accordingly, both international and national policy discourses position these competencies as core learning outcomes in tertiary mathematics education, highlighting their relevance in preparing undergraduates for STEM fields.

However, despite these policy aspirations, empirical evidence suggests a persistent gap between intended curriculum outcomes and the realities of undergraduate mathematics learning. Research consistently indicates that students transitioning into higher education are not always equipped with the conceptual and strategic resources necessary to meet such expectations (Adinda et al., 2021). Specifically, Mohd Nopiah et al. (2025) report many first-year undergraduates struggle to operate on unknowns, coordinate multiple representations, and select strategies for non-routine tasks. These difficulties directly reflect gaps in algebraic thinking, as students face challenges in recognizing patterns, manipulating symbols, and reasoning about relationships, and in flexible mathematical thinking, as they find it difficult to adapt strategies and represent problems in multiple ways. Moreover, classroom cultures oriented toward examination and over-reliance on rote learning often limit opportunities for students to develop algebraic and flexible mathematical thinking. Instructional strategies, particularly instructor-based instruction and learner-focused instruction, can address these gaps by guiding students to think systematically, apply concepts flexibly, and select appropriate strategies in non-routine tasks.

Algebraic thinking, defined as the ability to recognize patterns, manipulate symbols, and reason about mathematical relationships (Kaput, 2008; Radford, 2014), is essential to addressing these challenges. The development of algebraic thinking is strongly influenced by instructional strategies, as instructor-based instruction supports students in acquiring formal symbolic skills, while learner-focused instruction enables students to explore patterns, generalize relationships, and justify reasoning through active problem solving (Jamil et al., 2025; Lamon, 2020; Sun et al., 2023). Complementing this, flexible mathematical thinking refers to the capacity to adapt strategies, represent problems in multiple ways, and apply concepts in novel contexts (Heinze et al., 2009; Hickendorff et al., 2022). Such flexibility is more likely to emerge in learning environments that emphasize learner-focused instruction, where students are encouraged to compare solution methods, shift between representations, and make strategic decisions, rather than relying solely on fixed procedures demonstrated by the instructor (Low et al., 2019; Munaji et al., 2025; Star, 2018). Despite the recognized importance of these constructs, empirical evidence shows that many students' algebraic and flexible mathematical thinking remain underdeveloped, and the effectiveness of instructional strategies in addressing these gaps is not well understood (Lamon, 2020; Parviainen et al., 2024). Instructional strategies, particularly instructor-based instruction and learner-focused instruction, have the potential to address this issue by promoting systematic reasoning, strategic thinking, and flexible application of algebraic concepts (Chen et al., 2025; Rivers et al., 2020).

While existing studies have examined aspects of algebra learning and problem solving, many have focused on these constructs in isolation (Jamil et al., 2025; Pitta-Pantazi et al., 2019; Sibgatullin et al., 2022). Empirical research that integrates instructional strategies, algebraic thinking, and flexible mathematical thinking (Akdeniz, 2016; Castro-Alonso et al., 2021; Musengimana et al., 2025), particularly within the context of Malaysian higher education, remains limited. Addressing this gap, the present study examines the relationships among instructional strategies, students' algebraic thinking, and flexible mathematical thinking in Malaysian higher education.

RESEARCH OBJECTIVES AND QUESTIONS

This study's objectives are twofold. The first is to determine whether the instrument measuring instructional strategies, algebraic thinking, and flexible mathematical thinking among undergraduate students adequately represents the study constructs. The second is to establish whether the instrument fulfils the criteria for validity and reliability. In alignment with these objectives, the study addressed the following research questions:

1) Does the instrument measuring instructional strategies, algebraic thinking, and flexible mathematical thinking among undergraduate students adequately represent the study constructs?

2) Does the instrument fulfil the criteria for validity and reliability?

LITERATURE REVIEW

Instructional Strategies

Wider research indicates that instructional strategies are central to how undergraduate students engage with algebraic tasks requiring strategic reasoning and flexibility. Instructional strategies refer to deliberate, theory-informed pedagogical decisions by lecturers to organise content, structure learning activities, and mediate students' engagement with mathematical ideas (Akdeniz, 2016; Larson & Keiper, 2013). Such strategies shape cognitive and metacognitive processes, influencing how students interpret algebraic representations, select solution pathways, and evaluate the validity of their reasoning (Akdeniz, 2016; Lamon, 2020; Musengimana et al., 2025). Approaches range from instructor-focused strategies, such as explicit lecturing, step-by-step worked examples, direct explanation of symbolic procedures, and guided practice, which emphasise teacher control, modelling, and structured progression of content (Castro-Alonso et al., 2021; Parviainen et al., 2024). They also include learner-focused strategies, such as problem-based learning, inquiry-oriented tasks, and collaborative group work, in which students actively explore algebraic problems, discuss alternative solution methods, and justify their reasoning using multiple representations (symbolic, graphical, and numerical), which prioritise active participation, reflection, and autonomy (Bruner, 1960; Dick et al., 2015; Joyce et al., 2004; Merrill, 2013).

While instructor-focused strategies, including demonstration of standard algebraic solution methods, procedural drills, and scaffolded practice under lecturer guidance, are effective in supporting initial understanding of algebraic structures, reducing cognitive load, and improving procedural fluency, they may limit opportunities for students to independently construct meaning and explore alternative solution strategies (Mohd Nopiah et al., 2025). In contrast, learner-focused strategies, such as problem-based learning, collaborative problem solving, inquiry-oriented tasks, and the use of multiple representations (symbolic, graphical, and numerical), encourage students to generate, compare, and adapt solution strategies, thereby enhancing algebraic reasoning and flexible mathematical thinking, but may be less effective for students with weak prior knowledge if adequate scaffolding is not provided (Castro-Alonso et al., 2021).

Empirical studies suggest that while instructional strategies impact achievement and engagement, relatively fewer investigations have examined their role in developing higher-order outcomes such as algebraic thinking and flexible mathematical thinking (Castro-Alonso et al., 2021; Musengimana et al., 2025). Moreover, much of the existing research has been conducted at the school level or has focused on isolated instructional approaches (Lamon, 2020; Larson & Keiper, 2013; Musengimana et al., 2025), limiting insights into their combined effects at the undergraduate level. In summary, understanding instructional strategies, particularly the distinction between instructor-based instruction that emphasises structured guidance and modelling and learner-focused instruction that emphasises exploration, autonomy, and strategic decision making, is essential for promoting students' algebraic thinking and flexible mathematical thinking in higher education.

Algebraic Thinking

Kieran (2004) defined algebraic thinking as a process by which students are analysing relationships between quantities, noticing structures, studying changes, generalising, solving problems, modelling, justifying, proving, and predicting. Researchers have conceptualized algebraic thinking as involving generalization, abstraction, symbolic representation, modelling, and structural reasoning (Falkner et al., 1999; Kaput, 2008; Kieran, 1992, 1996, 2004; Lew, 2004; Lins, 1992). While these conceptualizations provide a robust theoretical foundation, much of the empirical work treats algebraic thinking as a static cognitive ability rather than a skill shaped dynamically by teachers' instructional practices (Radford, 2018; Sibgatullin et al., 2022; Steinweg et al., 2018).

To provide a more structured account, Kaput's (2008) framework identifies three interrelated strands of algebraic thinking, each representing a distinct aspect of how students reason algebraically. Generalised arithmetic involves recognising patterns and structures within arithmetic operations and constructing generalisations from these relationships (Davydov, 2008; Kaput, 2008; Radford, 2014). Functions focuses on understanding how quantities co-vary, reasoning about relationships between variables, and generalising patterns in both numerical and figural contexts (Blanton & Kaput, 2005; Kaput, 2008). Modelling emphasises using algebraic representations to describe, analyse, and generalise real-world or mathematical situations, progressing from situation-specific problem-solving to abstract and parameterised representations (Blanton & Kaput, 2005; Kaput, 2008). All of these strands capture the development of algebraic thinking from concrete arithmetic reasoning to flexible, functional, and representationally rich approaches.

Empirical studies indicate that many students struggle with algebraic expressions, variable notation, interpreting the equal sign, and transitioning from arithmetic to algebraic reasoning (Jupri et al., 2014; Loibl & Leuders, 2019; Tanisli & Ayber, 2017; Zayyadi et al., 2019). These difficulties are influenced by factors such as prior knowledge, scaffolding, and exposure to multiple representations, all of which are closely associated with teachers' instructional approach (Polya, 1945; Radford, 2008, 2018; Rivera, 2017; Torres et al., 2021). Many undergraduates tend to default to linear reasoning or rely on rhetorical explanations rather than fully engaging with functional or abstract thinking (Kieran, 1996; Lamon, 2020; Welder, 2012). This tendency is often linked to instructional environments that prioritise procedural fluency and examination of performance over conceptual exploration and strategic reasoning (Mohd Nopiah et al., 2025). Existing empirical studies frequently focus on identifying students' errors rather than examining how specific instructional strategies can address these difficulties (Kenney & Ntow, 2024; Mathaba et al., 2024; Pramesti & Retnawati, 2019). In summary, algebraic thinking is a complex skill that requires generalization, abstraction, and representational fluency, yet many undergraduates struggle to fully engage with these higher-order reasoning processes. Research evidence suggests that students' difficulties with algebraic thinking may be influenced by the types of instructional strategies employed in the mathematics classroom (Mohd Nopiah et al., 2025; Zakaria et al., 2019), with instructor-based and learner-focused approaches shaping how students engage with algebraic concepts, construct representations, and develop flexible reasoning skills.

Flexible Mathematical Thinking

Flexible mathematical thinking refers to the ability to select, adapt, and shift problem-solving strategies based on task demands, conceptual understanding, and representational considerations (Heinze et al., 2009; Hickendorff et al., 2022). It involves cognitive and metacognitive dimensions, including strategy repertoire, effectiveness, selection, and representational flexibility, allowing learners to interpret problems from multiple perspectives, generate novel solutions, and adapt approaches in real time (de Jong et al., 1998; Deliyianni et al., 2016). Heinze et al. (2009) distinguish between flexibility, which is the capacity to choose among multiple strategies, and adaptivity, which is the competence to select the most appropriate strategy for a given problem, highlighting that flexible mathematical thinking reflects the development of adaptive expertise. Similarly, theoretical frameworks, such as Siegler's (1996) dimensions of strategy development, emphasize that flexibility requires both cognitive diversity and metacognitive control. Although flexible mathematical thinking is widely acknowledged as critical for higher-order problem solving, empirical studies often emphasize outcome performance rather than the underlying instructional or cognitive conditions that enhance flexibility (Coppersmith, 2022; Huang et al., 2020; Patiño Jr, 2023). This limits understanding of how teaching practices can actively promote adaptive reasoning in mathematics.

Research indicates that many undergraduates struggle with non-routine problems due to limited strategic diversity, rigid thinking, and insufficient exposure to multiple representations (Andayani & Lathifah, 2019; Deliyianni et al., 2016; Liljedahl et al., 2016). Such difficulties may be linked to the instructional strategies employed in mathematics classrooms. Learner-focused strategies, including problem-based learning, inquiry-oriented tasks, and collaborative exploration, promote students' engagement with multiple solution paths, encourage reflection on alternative strategies, and foster cognitive and metacognitive growth (Castro-Alonso et al., 2021; Star & Rittle-Johnson, 2008). Instructor-focused strategies, such as step-by-step worked examples, scaffolded practice, and corrective feedback, provide essential guidance for novices and reduce cognitive load, supporting initial understanding of algebraic concepts (Mohd Nopiah et al., 2025). Both approaches are complementary: learner-focused strategies enhance independent strategic thinking, while instructor-focused strategies ensure foundational understanding, making the development of flexible mathematical thinking dependent on the careful design and balance of instructional strategies. These strategies mediate the activation, selection, and adaptation of problem-solving approaches, establishing their central role as the independent variable in this study.

To conclude, developing flexible mathematical thinking requires not only procedural fluency or exposure to multiple strategies but also cognitive flexibility, including real-time strategy adjustment, schema-based reasoning, representational adaptability, all of which are shaped by instructional approaches (Huang et al., 2020; Loibl et al., 2020). Integrating both learner-focused and instructor-based strategies, such as scaffolded guidance for foundational understanding alongside opportunities for exploration, reflection, and strategy transfer, allows teaching practices to support the development of adaptive expertise. This approach helps undergraduates to navigate non-routine mathematical tasks effectively and strategically. This reflects the critical role of pedagogy in supporting higher-order problem-solving skills and adaptive reasoning in higher education mathematics.

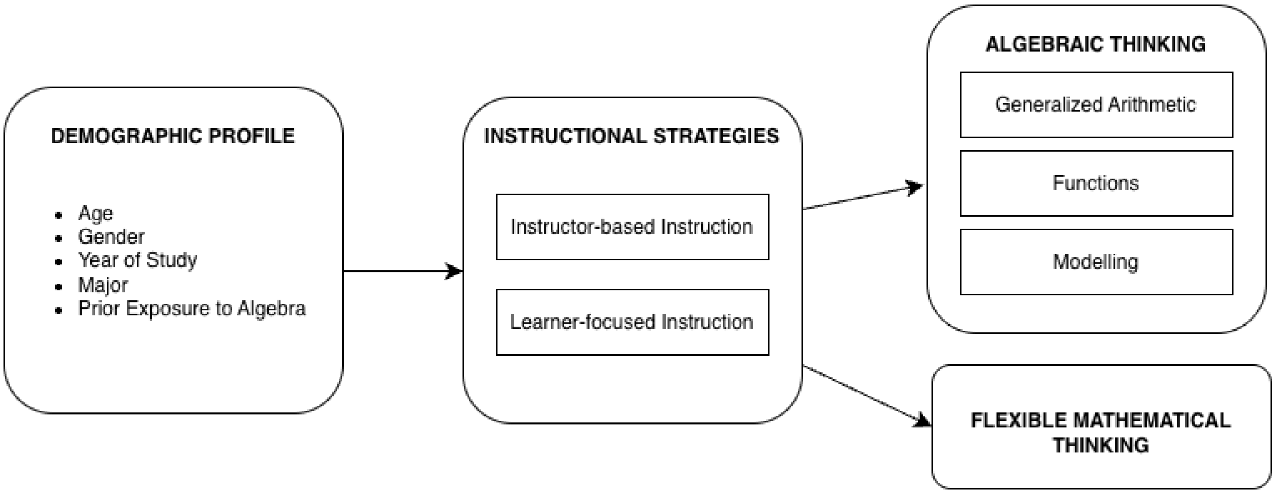
METHODOLOGY

Research Design

This study is situated within the positivist quantitative research paradigm and employs a cross-sectional correlational survey design. The primary purpose is to examine the reliability and validity of the survey instrument in measuring instructional strategies, algebraic thinking, and flexible mathematical thinking among undergraduate students enrolled in mathematics-related programmes. The conceptual model of the study is presented in Figure 1.

Figure 1

Conceptual Model



Sample

The sample comprised thirty-three (N = 33) undergraduate students enrolled in mathematics-related programmes at a Malaysian public higher education institution who had completed introductory university-level courses in Analytic Geometry and Calculus I during Semester 1. They represented students from all four years of study (Year 1–4) to ensure sample diversity and representativeness. The respondents were required to provide voluntary informed consent to participate in the survey. Students without sufficient exposure to algebraic concepts were excluded to ensure validity of responses regarding instructional strategies, algebraic thinking, and flexible mathematical thinking. Academic and topic-specific characteristics focused on their achievement and prior learning experiences in algebra to ensure they could provide informed and meaningful responses on the constructs of instructional strategies, algebraic thinking, and flexible mathematical thinking. Given the exploratory nature of this pilot study, the sample size was considered adequate for preliminary psychometric analysis using Partial Least Squares Structural Equation Modelling (PLS-SEM), which is suitable for small samples and early-stage instrument development. Accordingly, the reliability indicators (Cronbach's alpha and composite reliability) and validity assessments obtained from this sample are interpreted as preliminary evidence intended to inform future large-scale validation studies. The inclusion of Year 1 students is justified as these students had already studied foundational algebraic and analytic concepts through their core mathematics courses, ensuring they could provide informed and meaningful responses regarding the constructs under investigation.

Measures and Covariates

The primary measures for this study focused on three major constructs, namely (1) instructional strategies which included two factors (instructor-based instruction and learner-focused instruction) that captured different approaches to teaching mathematics; (2) algebraic thinking, which comprised the factors of functions, generalized arithmetic, and modelling that assessed students' ability to reason and operate with algebraic concepts; and (3) flexible mathematical thinking, which measured students' capacity to apply mathematical reasoning in varied and non-routine contexts. In addition to the primary constructs, covariate data were collected to provide context and account for individual differences. These included demographic information such as students' age, gender, year of study, and prior exposure to algebra, which could influence students' mathematical reasoning and engagement with instructional strategies.

Instrument

The study employed a validated and adapted questionnaire consisting of 47 items designed to measure instructional strategies, algebraic thinking, and flexible mathematical thinking. Items drew on the work of six established studies (Alhunaini et al., 2022; Barak & Levenberg, 2016; Eristi & Akdeniz, 2012; Jahudin & Siew, 2023; Kaput, 2008; Martin & Rubin, 1995; Ralston, 2013) and were refined to suit the undergraduate algebra context. Items were rated on a five-point Likert scale ranging from "Strongly Disagree" (1) to "Strongly Agree" (5). Psychometric properties were evaluated using internal consistency coefficients for composite scales, as well as convergent and discriminant validity for latent constructs.

Items measuring instructor-based instruction and learner-focused instruction were adapted from Eristi and Akdeniz (2012), ensuring alignment with established frameworks for teaching strategies in mathematics. Algebraic thinking was divided into generalized arithmetic, functions, and modelling. Items for functions were based on Kaput (2008) and Ralston (2013), items for Modelling were adapted from Alhunaini (2022) and Kaput (2008), and items for Generalized Arithmetic were drawn from Alhunaini (2022), Jahudin and Siew (2023), and Kaput (2008) to capture students' ability to generalize, reason with unknowns, and coordinate multiple representations. Items measuring Flexible Mathematical Thinking were referenced from Barak and Levenberg (2016) and Martin and Rubin (1995) to assess students' capacity for strategy selection, adaptation, and representational flexibility in problem solving. Table 1 presents the survey instrument used in this study, including all constructs, their operational definitions, item codes, and the full text of each item.

Table 1

Survey Instrument Items and Construct Definitions

Construct	Definition	Item Code	Item Text
Instructor-based Instruction	Teacher-centered strategies where the lecturer primarily directs learning, emphasizing structured guidance, uniform instruction, and corrective feedback (Arends & Castle, 2002; Jonassen et al., 1990; Molenda & Subramony, 2020)	IF1	The lecturer considers all students and their individual differences.
		IF2	The lecturer assigns the same duties and responsibilities to students.
		IF3	The lecturer corrects deficiencies and mistakes in students' work.
		IF4	The lecturer has students take notes during lessons.
		IF5	The lecturer ensures the whole class acquires all curriculum gains.

Learner-focused Instruction	Student-centered strategies emphasizing participation, discussion, reflection, and self-directed decision-making (Arends & Castle, 2002; Molenda & Subramony, 2020)	LF1	The lecturer uses metaphors in instruction.
		LF2	The lecturer chooses real-life examples considering students' characteristics.
		LF3	The lecturer explains instructional goals to students.
		LF4	The lecturer uses cooperation and discussion rather than purely lecturing.
		LF5	The lecturer connects students' background knowledge to new learning.
		LF6	The lecturer helps students determine content for independent study.
		LF7	The lecturer encourages students to ask questions and express views.
		LF8	The lecturer facilitates student discussion of problem-solving strategies.
		LF9	The lecturer allows students to choose learning preferences based on interests.
Generalized Arithmetic	Ability to recognize patterns, generalize arithmetic relationships, and apply arithmetic operations using variables and symbolic reasoning (Davydov, 2008; Kaput, 2008; Radford, 2014)	GA1	I can perform calculations correctly when all numbers in a problem are provided.
		GA2	I can determine whether two sides of an equation are equal by calculating them.
		GA3	I can solve simple problems by trying different possible values for unknowns.
		GA4	I can identify patterns in small and straightforward calculations.
		GA5	I can extend a pattern I notice in calculations to new but similar problems.
		GA6	I can use relational thinking to check if two expressions are equal.
		GA7	I can solve for unknowns using basic equation properties, such as inverse operations.
		GA8	I can recognize patterns in calculations and apply them to problems with similar structure.
		GA9	I can decide whether two expressions are equal without computing every value.
Functions	Ability to understand, represent, evaluate, and	F1	I can identify simple patterns in number sequences.

	interpret algebraic functions and their transformations (Blanton & Kaput, 2005; Kaput, 2008)	F2	I can notice when two quantities increase or decrease together in simple examples.
		F3	I can describe a pattern using words, such as "adds three each time."
		F4	I can match two sets of quantities that change together in straightforward situations.
		F5	I can find recursive rules for number or object patterns.
		F6	I can determine the relationship between variables in simple functional situations.
		F7	I can use tables to represent relationships between changing quantities.
		F8	I can predict the next term in a pattern using a rule I discovered.
		F9	I can translate a pattern into a formula using letters for unknowns.
Modelling	Ability to translate real-world situations into algebraic representations and use mathematical relationships to analyze and solve problems (Blanton & Kaput, 2005; Kaput, 2008)	M1	I can identify important information in a real-life situation.
		M2	I can explain a mathematical idea using simple statements.
		M3	I can convert a basic real-world problem into a simple mathematical expression.
		M4	I can read a simple graph and understand what it represents.
		M5	I can recognize patterns in a situation before creating a model.
Flexible Mathematical Thinking	Ability to recognize problem structures, generate and compare multiple solution strategies, and adapt approaches for efficient solutions (Liljedahl et al., 2016; Rittle-Johnson et al., 2001)	FMT1	I notice when a problem has features that allow a simpler way to solve it.
		FMT2	I can choose a strategy that reduces unnecessary steps.
		FMT3	I can think of more than one way to solve a problem.
		FMT4	I recognize when a familiar strategy will make a problem easier.
		FMT5	I identify opportunities to use shortcuts without affecting accuracy.
		FMT6	I adjust my strategy when I realize the problem has unique features.

	FMT7	I choose the approach that is most efficient for the problem at hand.
	FMT8	I consider how the structure of a problem influences my choice of method.
	FMT9	I compare several strategies before deciding which one to use.
	FMT10	I refine my method when I notice that my initial approach is not optimal.

The questionnaire underwent expert review by five mathematics education and curriculum specialists to ensure clarity, construct coverage, and cultural appropriateness. It was then piloted with 33 students to assess internal consistency, clarity, and preliminary construct validity. The online survey method ensured confidentiality, flexibility, and efficient administration of data collection.

DATA COLLECTION AND DATA ANALYSIS

Data Collection

Data were collected via a structured online survey administered to participants. The survey comprised four sections: (1) demographic information, (2) instructional strategies, (3) algebraic thinking, and (4) flexible mathematical thinking. The instructional strategies section included two sub-constructs: Instructor-based Instruction (5 items) and Learner-focused Instruction (9 items). The algebraic thinking section consisted of three sub-constructs: Generalized Arithmetic (9 items), Functions (9 items), and Modelling (5 items). Finally, the flexible mathematical thinking section included 10 items.

The survey links were shared via email to the respondents, who were selected purposively to ensure relevance to the study. Data collection was conducted over a period of three weeks, with respondents provided clear instructions and sufficient time to complete the questionnaire. Approval for the study was obtained from the Kulliyyah of Education, International Islamic University Malaysia (IIUM). In addition, permission to collect data from students in the mathematics courses was obtained from the Kulliyyah of Sciences, IIUM. All respondents provided informed consent and were clearly informed about the study’s purpose, procedures, potential risks, and benefits. Participation was entirely voluntary, and students had the right to withdraw at any stage without penalty. Confidentiality was maintained by removing personal identifiers and using pseudonyms in all reporting. All procedures adhered to ethical standards in social science research (Creswell & Creswell, 2022; Creswell & Poth, 2018; Denzin & Lincoln, 1994).

Data Analysis

Preliminary analysis using SPSS version 30 examined respondents’ demographic profiles and screened for missing data, and outliers. Descriptive statistics, including means, standard deviations, frequencies, and percentages, were computed to summarise the sample characteristics and ensure data readiness for structural modelling (Chin, 1998; Gefen et al., 2000). Structural Equation Modelling (SEM) was performed using SmartPLS version 4 to evaluate the measurement model. PLS-SEM was selected due to its suitability for complex models with multiple latent constructs, its robustness with moderate sample sizes, and its ability to handle reflective constructs (Hair et al., 2014; Henseler et al., 2009). Preliminary analysis was conducted using SPSS version 30 to examine respondents’ demographic profiles and screen for missing data and outliers. Since PLS-SEM is a non-parametric technique, it does not assume normality, making it suitable for this pilot study (N = 33) where the primary goal was to evaluate instrument reliability and validity rather than test hypotheses (Hair et al., 2014; Henseler et al., 2009). Extreme skewness or kurtosis may affect bootstrap significance, but this is not critical at the preliminary stage (Chin, 1998; Gefen et al., 2000). Descriptive statistics, including means, standard deviations, frequencies, and percentages, were computed to summarise the sample characteristics and ensure data readiness for structural modelling (Chin, 1998; Gefen et al., 2000).

Then, structural Equation Modelling (SEM) was performed using SmartPLS version 4 to evaluate the measurement model. PLS-SEM was selected due to its suitability for complex models with multiple latent constructs, its robustness with moderate sample sizes, and its ability to handle reflective constructs (Hair et al., 2014; Henseler et al., 2009). The measurement model was assessed for reliability and validity. Reliability, which evaluates the consistency and dependability of the instrument, was examined using Cronbach's alpha and composite reliability (CR), with thresholds of ≥0.70 indicating acceptable internal consistency (Fornell & Larcker, 1981; Hair et al., 2010; Sekaran & Bougie, 2016). Convergent validity, which assesses whether items adequately measure the intended construct, was evaluated using factor loadings (≥0.50), CR (≥0.70), and Average Variance Extracted (AVE ≥0.50) (Chin, 1998; Hair et al., 2010). Discriminant validity, which ensures that constructs are distinct from one another, was assessed using the Fornell–Larcker criterion, cross-loadings, and the Heterotrait–Monotrait (HTMT) ratio (<0.90) (Henseler et al., 2015; Kline, 2011). Indicators were retained only if they met these thresholds, ensuring construct consistency, unidimensionality, and clear distinction between constructs. Table 2 presents the criteria for assessing validity and reliability results for the measurement model.

Table 2

Criteria for Assessing Reliability and Validity of the Measurement Model

Assessment Aspect	Indicator	Recommended Threshold	Interpretation	References
Construct Reliability	Composite Reliability (CR)	≥ 0.70 indicates acceptable reliability; < 0.60 indicates weak reliability	Demonstrates internal consistency of the construct	Fornell & Larcker (1981); Hair et al. (2010); Kline (2011)
Convergent Validity	Average Variance Extracted (AVE)	≥ 0.50	Indicates that the construct explains sufficient variance in its indicators	Hair et al., (2010, 2014, 2016); Chin 1998); Hulland, (1999)
	Standardised Factor Loadings	≥ 0.50 (preferably ≥ 0.70)	Confirms that items adequately represent the intended construct	Fornell & Larcker (1981); Hair et al., (2010)
Discriminant Validity	Cross-loadings	Indicator loading on its own construct is greater than loadings on other constructs	Ensures each indicator is most strongly associated with its intended construct	Hair et al., (2010)
	Fornell-Larcker Criterion (Square Root of AVE)	Square root of AVE should exceed correlation with other constructs	Demonstrates that each construct is more strongly related to its	Fornell & Larcker (1981)

			own indicators than to other constructs	
	HTMT (Heterotrait-Monotrait Ratio)	<0.85 (conservative) <0.90 (liberal) All values below 1 confirm discriminant validity	Confirms constructs are empirically distinct	Kline, (2011); Gold, (2001); Henseler, (2015)

RESULTS

Profile of the Respondents

Table 3 shows the characteristics of the 33 respondents involved in the pilot study by age, gender, year of study, major and exposure to algebra.

Table 3

Respondents' Demographic Profile (N=33)

Variable	N	%
Age		
■ Below 20	11	33.3
■ 20 – 22	20	60.6
■ 23 – 25	1	3
■ >26	1	3
Gender		
■ Female	11	33.3
■ Male	22	66.7
Year of Study		
■ Year 1	13	39.4
■ Year 2	6	18.2
■ Year 3	6	18.2
■ Year 4	8	24.2
Major		
■ Pure Mathematics	4	12.1
■ Statistics	9	27.3
■ Computational Mathematics	6	18.2
■ Financial Mathematics	7	21.2
■ Operational Research	7	21.2
Prior Exposure to Algebra		
■ Extensive (advanced high school/pre-university mathematics)	14	42.4
■ Moderate (standard high school mathematics)	19	57.6
■ Minimal (basic mathematics only)	0	0.00

Based on Table 1, the distribution of respondents by gender shows that 33.3% were female, while 66.7% were male. The majority of respondents were aged between 20 and 22 years (60.6%), followed by those below 20 years old (33.3%). Only a small proportion of respondents were aged 23–25 years and above 26 years, each accounting for 3.0%. In terms of year of study, most respondents were in Year 1 (39.4%), followed by Year 4 (24.2%), while Years 2 and 3 each represented 18.2% of the sample. With regard to academic major, the largest proportion of respondents were majoring in Statistics (27.3%), followed by Financial Mathematics and Operational Research (both 21.2%), Computational Mathematics (18.2%), and Pure Mathematics (12.1%). Regarding prior exposure to algebra, the majority of respondents reported moderate exposure (57.6%), while 42.4% reported extensive exposure. None of the respondents reported minimal exposure. Overall, the demographic profile indicates that the respondents were predominantly male, aged 20–22 years, primarily in Year 1, majoring in Statistics, and possessing moderate prior exposure to algebra or advanced mathematics.

Findings of Measurement Model Analysis

Construct Reliability

To establish whether the items in the study could adequately measure related constructs in scores, the reliability of the measurement model was assessed using internal consistency reliability, specifically through Cronbach's alpha and composite reliability (CR). Internal consistency reliability examines the extent to which items within a construct consistently measure the same underlying concept (Hair et al., 2016). Cronbach's alpha values exceeding 0.70 indicate acceptable reliability of the measurement items (Nunnally & Bernstein, 1994), while composite reliability values of 0.70 or higher reflect satisfactory internal consistency for established studies, with values between 0.60 and 0.70 considered acceptable in exploratory research (Fornell & Larcker, 1981; Nunnally & Bernstein, 1994). In addition, CR values greater than 0.70 are recommended to ensure adequate internal consistency reliability (Hair et al., 2016). As shown in Table 4, the Cronbach's alpha values ranged from 0.78 to 0.93, and composite reliability values ranged from 0.88 to 0.94, indicating that the instruments demonstrated strong and consistent measurement of the intended constructs.

Table 4

Findings of Internal Consistency Reliability

Constructs	Item	Internal Consistency Reliability	
		Cronbach Alpha *(α >0.70)*	Composite Reliability *(CR>0.70)*
Instructional Strategies			
■ Instructor-based Instruction	5	0.78	0.84
■ Learner-focused Instruction	9	0.85	0.88
Algebraic Thinking			
■ Generalized Arithmetic	9	0.90	0.92
■ Functions	9	0.93	0.94
■ Modelling	5	0.90	0.93
Flexible Mathematical Thinking	10	0.90	0.92

Note. This table was adapted from Hair et al. (2017).

Convergent Validity

In response to the second research question, the results measures how well an item measures related construct in a study was evaluated through convergent validity. Convergent validity examines the extent to which multiple indicators of a construct share a high proportion of variance and effectively measure the same underlying concept (Hair et al., 2016). In this study, convergent validity was assessed using three criteria: outer loadings, composite reliability (CR), and average variance extracted (AVE) (Hair et al., 2014, 2016). Outer loading values of 0.708 or higher were considered acceptable, as this threshold indicates that an indicator explains at least 50% of the variance of its construct, corresponding to the minimum AVE requirement of 0.50 (Hair et al., 2016). Indicators with outer loadings between 0.40 and 0.70 were carefully examined and retained only if their removal did not result in substantial improvements in AVE or CR. Additionally, outer loading values exceeding 0.50 were deemed acceptable, as such values indicate adequate indicator reliability (Chin, 1998; Hulland, 1999). Although modern PLS-SEM standards often recommend outer loadings ≥

0.708 (Hair et al., 2016), this pilot study adopted a more lenient threshold of ≥ 0.50 (Chin, 1998; Hulland, 1999) to allow for preliminary assessment of items under small sample conditions. Items not meeting this threshold were removed, and the remaining items provide indicative evidence of reliability and construct validity to inform future large-scale validation. Overall, the convergent validity results demonstrate that the measurement items adequately represented their respective latent constructs. Table 5 summarises the findings of convergent validity.

Table 5

Findings of Convergent Validity

Construct	Items	Loading *(>0.5)*	AVE *(>0.5)*	CR *(>0.7)*
Instructor-based Instruction	IF1	0.62	0.52	0.84
	IF2	0.78		
	IF3	0.70		
	IF4	0.70		
	IF5	0.79		
Learner-focused Instruction	LF1	0.54	0.55	0.88
	LF2	0.75		
	LF3	0.63		
	LF4	0.58		
	LF5	0.70		
	LF6	0.67		
	LF7	0.83		
	LF8	0.74		
	LF9	0.54		
Generalized Arithmetic	GA1	0.72	0.56	0.92
	GA2	0.89		
	GA3	0.91		
	GA4	0.88		
	GA5	0.66		
	GA6	0.80		
	GA7	0.70		
	GA8	0.69		
	GA9	0.32		
Functions	F1	0.79	0.65	0.94
	F2	0.77		
	F3	0.88		
	F4	0.89		
	F5	0.82		
	F6	0.88		
	F7	0.70		
	F8	0.72		
	F9	0.80		
Modelling	M1	0.88	0.72	0.93
	M2	0.86		
	M3	0.88		
	M4	0.85		
	M5	0.78		
Flexible Mathematical Thinking	FMT1	0.83	0.55	0.92
	FMT2	0.71		
	FMT3	0.40		

FMT4	0.89
FMT5	0.65
FMT6	0.82
FMT7	0.86
FMT8	0.87
FMT9	0.48
FMT10	0.76

In this study, certain items were removed because they did not meet the minimum thresholds, with AVE required to be greater than 0.50 (Bartlett et al., 2001) and CR required to exceed 0.70 (Hair et al., 2016). All remaining items satisfied these criteria, except for GA9, FMT3, and FMT9, which were excluded due to insufficient outer loadings. Specifically, GA9 *("I can decide whether two expressions are equal without computing every value")* may have failed because students at this level might lack the abstract reasoning skills required. FMT3 *("I can think of more than one way to solve a problem")* was likely too advanced or open-ended, causing inconsistent responses. FMT9 *("I compare several strategies before deciding which one to use")* may have been confusing or double-barreled, combining comparison and selection, which increased variability in answers.

Next, discriminant validity analysis was conducted to evaluate the extent to which each construct is distinct from the other constructs. This assessment examines the degree of correlation between constructs and the ability of items to adequately represent a single construct (Hair et al., 2016). In this study, three methods were used to assess discriminant validity: (1) cross-loadings, (2) Fornell–Larcker criterion, and (3) Heterotrait–Monotrait ratio (HTMT).

Cross-loading analysis examines whether each item loads more strongly on its intended construct than on any other construct (Hair et al., 2016). This assessment ensures that each indicator predominantly represents its latent variable, reducing the risk of multicollinearity among constructs (Chin, 1998; Fornell & Larcker, 1981; Vinzi et al., 2022). If an item's loading on another construct exceeds its loading on the intended construct, it indicates a potential problem with discriminant validity (Hair et al., 2016). The results of this study showed that each item's loading on its respective construct was higher than on any other construct, providing evidence that the measurement model exhibited satisfactory discriminant validity.

The Fornell–Larcker criterion compares the square root of a construct's AVE with its correlations with other constructs, with the AVE square root expected to be larger than all inter-construct correlations (Hair et al., 2016). This criterion ensures that a latent variable explains its own indicators better than it shares variance with other constructs. In this study, after removing items that did not meet the minimum outer loading thresholds, the square roots of the AVEs were higher than the correlations with all other constructs (Table 6). These results confirm that the Fornell–Larcker criterion was satisfied, supporting the discriminant validity of the measurement model and addressing the research questions regarding construct validity.

Table 6

Findings of Fornell–Larcker criterion Analysis

	1	2	3	4	5	6
1	**0.80**					
2	0.57	**0.81**				
3	0.70	0.66	**0.79**			
4	0.67	0.44	0.53	**0.73**		
5	0.60	0.35	0.60	0.70	**0.88**	
6	0.69	0.56	0.74	0.56	0.55	**0.85**

Notes: 1 = Flexible Mathematical Thinking; 2 = Functions; 3 = Generalized Arithmetic; 4 = Instructor-based Instruction; 5 = Learner-focused Instruction; 6 = Modelling

The Heterotrait-Monotrait Ratio (HTMT) was used to assess discriminant validity, with recommended thresholds of 0.85 (Kline, 2011) or 0.90 (Gold et al., 2001). Statistically, HTMT can be tested using the null hypothesis (H_0: HTMT < 1) against the alternative (H_1: HTMT ≥ 1) (Henseler et al., 2015). If the 95% confidence interval of the HTMT includes the value 1, discriminant validity is not established. As shown in Table 7 and Table 8, all HTMT values for the constructs were below 1, and the lower and upper bounds of the 95% bootstrap confidence intervals were all below 1.0. This confirms the presence of discriminant validity in the measurement model. The outcomes of the model are illustrated in Figure 2.

Table 7

Findings of Heterotrait-Monotrait (HTMT) Analysis

	1	**2**	**3**	**4**	**5**	**6**
1						
2	0.57					
3	0.71	0.72				
4	0.76	0.45	0.60			
5	0.66	0.37	0.57	0.76		
6	0.74	0.59	0.78	0.61	0.54	

Note. Diagonals represent the square root of the average variance extracted, while the other entries represent the squared Correlations Coefficients. 1 = Flexible Mathematical Thinking; 2 = Functions; 3 = Generalized Arithmetic; 4 = Instructor-based Instruction; 5 = Learner-focused Instruction; 6 = Modelling

Table 8

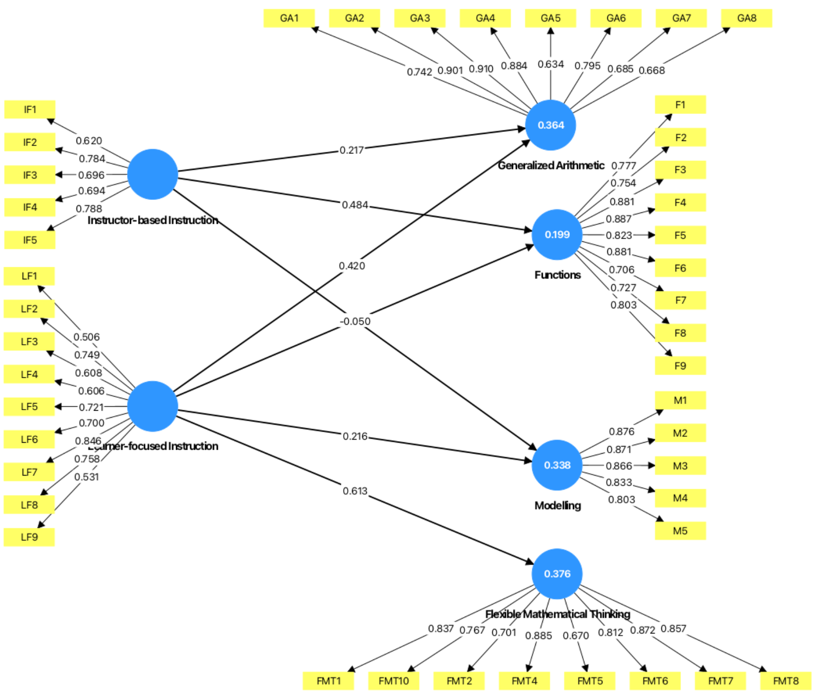
Findings of Confidence Bias

	Original sample (O)	**Sample mean (M)**	**Bias**	**2.5%**	**95%**
Functions <-> Flexible Mathematical Thinking	0.57	0.59	0.02	0.30	0.81
Generalized Arithmetic <-> Flexible Mathematical Thinking	0.71	0.72	0.01	0.48	0.86
Generalized Arithmetic <-> Functions	0.72	0.72	0.00	0.48	0.88
Instructor-based Instruction <-> Flexible Mathematical Thinking	0.76	0.78	0.02	0.56	0.93
Instructor-based Instruction <-> Functions	0.45	0.52	0.07	0.26	0.63
Instructor-based Instruction <-> Generalized Arithmetic	0.60	0.64	0.04	0.34	0.78
Learner-focused Instruction <-> Flexible Mathematical Thinking	0.66	0.67	0.01	0.46	0.84

Learner-focused Instruction <-> Functions	0.37	0.46	0.09	0.25	0.44
Learner-focused Instruction <-> Generalized Arithmetic	0.57	0.62	0.04	0.38	0.71
Learner-focused Instruction <-> Instructor-based Instruction	0.88	0.89	0.01	0.75	0.91
Modelling <-> Flexible Mathematical Thinking	0.74	0.74	0.00	0.49	0.90
Modelling <-> Functions	0.59	0.63	0.04	0.35	0.77
Modelling <-> Generalized Arithmetic	0.78	0.80	0.03	0.58	0.91
Modelling <-> Instructor-based Instruction	0.61	0.66	0.05	0.44	0.79
Modelling <-> Learner-focused Instruction	0.54	0.60	0.06	0.36	0.68

Figure 2

SmartPLS Algorithm Value of the Measurement Model



Common Method Bias (CMB) Assessment

Since the data for both the independent variable (Instructional Strategies) and dependent variables (Algebraic Thinking and Flexible Mathematical Thinking) were collected from the same respondents at the same time using the same Likert scale, there is a risk of Common Method Variance (CMV) inflating correlations between constructs (Jarvis et al., 2003). To assess this risk, Full Collinearity Variance Inflation Factors (VIFs) were calculated for all constructs. As shown in Table 9, all VIF values were below the recommended threshold of 3.3 (Kock, 2015), indicating that CMV is not likely to be a significant threat to the measurement model.

Table 9

Findings of Variance Inflation Factors

	Structural Path	VIF
1)	Instructor-based Instruction -> Functions	2.47
2)	Instructor-based Instruction -> Generalized Arithmetic	2.47
3)	Instructor-based Instruction -> Modelling	2.47
4)	Learner-focused Instruction -> Flexible Mathematical Thinking	1.00
5)	Learner-focused Instruction -> Functions	2.47
6)	Learner-focused Instruction -> Generalized Arithmetic	2.47
7)	Learner-focused Instruction -> Modelling	2.47

DISCUSSION

PLS-SEM was employed to evaluate the measurement model, particularly internal consistency reliability and construct validity in this study as it allows for the analysis of complex structural equation models containing multiple constructs and indicators (Hair et al., 2016; Urbach & Ahlemann, 2010). This method was chosen due to its strong predictive and explanatory capabilities for the target constructs (Hair et al., 2016). Moreover, PLS-SEM offers several advantages over other analytical techniques. The results obtained relate to the reliability assessment, which ensures consistency across items measuring the same construct and determines whether the items accurately capture the intended concept (Hair et al., 2016).

The study considered composite reliability (CR) values to evaluate internal consistency, with a threshold of >0.7 indicating adequate reliability (Gefen et al., 2000; Hair et al., 2016). Similarly, Cronbach's Alpha values exceeding 0.7 were examined to assess the reliability of the items for each construct (Nunnally & Bernstein, 1994). The results revealed that CR values for all constructs ranged from 0.88 to 0.94, while Cronbach's Alpha values ranged from 0.78 to 0.93. These findings confirm that both CR and Cronbach's Alpha values were satisfactory, demonstrating that all six main constructs in the study exhibited high internal consistency reliability (Gefen et al., 2000; Nunnally & Bernstein, 1994). This indicates that the constructs of algebraic thinking, instructional strategies, and flexible mathematical thinking are well-represented by the measurement items.

Convergent validity was evaluated using CFA, which assesses the extent to which multiple items effectively measure the same construct (Hair et al., 2016). Three criteria were applied: i) outer loading > 0.5, ii) CR > 0.7, and iii) AVE > 0.5. The outer loading threshold followed Chin (1998) and Hulland (1999), who suggested that items exceeding 0.5 are adequate indicators. Based on this analysis, items GA9, FMT3, and FMT9 were removed due to low loadings, while the remaining items exceeded the minimum criteria, confirming satisfactory convergent validity (Bartlett, Kotrlik, & Higgins, 2001; Hair et al., 2016).

Discriminant Validity

Discriminant validity was assessed using cross-loadings, the Fornell–Larcker criterion, and the HTMT ratio. Cross-loading results indicated that all items loaded more strongly on their intended constructs than on others, showing no significant multicollinearity (Chin, 1998; Vinzi et al., 2022). The Fornell–Larcker criterion revealed that the square root of each construct's AVE was higher than its correlations with other constructs, supporting discriminant validity. Additionally, HTMT values were all below 0.85, indicating that the constructs were sufficiently distinct (Gold et al., 2001; Henseler et al., 2009; Kline, 2011). These findings demonstrate that the measurement model is both reliable and valid for assessing algebraic thinking, instructional strategies, and flexible mathematical thinking. Full collinearity VIF values were examined to assess potential common method bias, with all VIFs below the recommended threshold of 3.3 (ranging 1.00–2.47) (Kock, 2015). This indicates that multicollinearity among constructs is not likely to have substantially biased the results.

However, the pilot sample size (N = 33) and unbalanced gender distribution may limit the generalisability of the findings. With a small sample, standard errors may be inflated, which can affect the stability of estimated outer loadings and path coefficients. Although the sample size meets the minimum requirements commonly applied in PLS-SEM pilot studies, including the 10-times rule (Barclay et al., 1995), the results should be interpreted with caution. For the main study, a larger sample (N ≥ 150) is recommended to ensure stable factor loadings and significant path coefficients, and stratified random sampling should replace purposive sampling to better reflect the population and reduce bias. Accordingly, the reported reliability and validity evidence is considered preliminary. Future studies should employ larger and more balanced samples to enhance estimation stability, improve measurement precision, and allow for more robust assessment of structural relationships.

In addition to sample-related limitations, certain measurement items may reflect overlapping conceptual boundaries between procedural performance and higher-order algebraic thinking. For instance, Item GA7 emphasises solving for unknowns using basic equation properties, which may capture procedural execution rather than generalized arithmetic thinking. While such skills are foundational, this item may reflect "doing" mathematics more than underlying generalisation processes. Conversely, Item F3, which assesses the ability to describe patterns using words, appears to strongly represent verbal representation, a key dimension of algebraic thinking. This imbalance highlights the need for further refinement of item wording to ensure clearer alignment between theoretical constructs and their operationalisation in future validation studies.

It is also noted that the Average Variance Extracted (AVE) for the Instructor-based Instruction construct is 0.52, just above the recommended threshold of 0.50. While this meets the minimum criterion for convergent validity (Hair et al., 2016; Chin, 1998), it indicates that the items explain only slightly more variance than the measurement error. This suggests that the items for this construct are not strongly converging, and future instrument refinement may focus on improving item alignment and clarity to enhance convergence for Instructor-based Instruction.

CONCLUSION

This study focused on three main constructs: instructional strategies, algebraic thinking, and flexible mathematical thinking. Instructional strategies were further divided into instructor-based instruction and learner-focused instruction. Algebraic thinking included three sub-constructs: generalized arithmetic, functions, and modelling, while flexible mathematical thinking encompassed the capacity to adapt strategies, shift representations, and apply concepts in novel contexts. The primary aim of the study was to evaluate the reliability and validity of a survey instrument designed to measure these constructs among undergraduate mathematics students.

The findings from PLS-SEM analysis indicate that the survey instrument demonstrated high reliability and validity, with CR values ranging from 0.88 to 0.94 and Cronbach's Alpha values from 0.78 to 0.93, confirming strong internal consistency for all constructs and sub-constructs. Convergent validity (AVE 0.52–0.72) and discriminant validity (cross-loadings, Fornell–Larcker, HTMT < 0.85, and 95% bootstrap confidence intervals with lower and upper bounds below 1.0) further showed that the items effectively measured their intended constructs. Full Collinearity VIF values below 3.3 (ranging 1.00–2.47) (Kock, 2015) indicate minimal risk of common method bias, supporting the instrument's suitability for examining instructional strategies, algebraic thinking, and flexible mathematical thinking among undergraduate mathematics students.

The results have significant implications for tertiary mathematics education in Malaysia. The results provide information on the structure and measurement of instructional strategies, algebraic thinking, and flexible mathematical thinking (Arends & Castle, 2002; Kaput, 2008; Radford, 2014).

The validated survey instrument can serve as a tool for future research and pedagogical evaluation, offering guidance for developing similar measurement tools in related studies. From a policy and curriculum perspective, these findings can guide the Ministry of Education Malaysia (MOE) and higher education institutions in designing and refining mathematics curricula that support algebraic thinking and flexible application of algebraic concepts.

Overall, this pilot study establishes a reliable and valid measurement foundation for assessing instructional strategies, algebraic thinking, and flexible mathematical thinking in higher education mathematics contexts. Future research should extend this work by applying the instrument to larger samples and examining structural relationships or educational outcomes beyond the scope of the present study.

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