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TEMPORARY *WĀQF* AND PERPETUAL BENEFIT: A MATHEMATICAL PROOF

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ABSTRACT

Temporary wāqf (Islāmic pious foundation) possesses great unrealized potential of preserving maslahah (public interest), but to date, has not been given the attention it deserves. Using mathematical representations, a first in the $w\bar{a}qf$ literature, this paper seeks to prove that temporary $w\bar{a}qf$ can still provide a perpetual benefit similar to that of permanent wāqf. Logical mathematical derivations suggest that so long as large enough number of similar mauque fs (corpus of waqf) are endowed at different periods in time for similar purposes, the benefit derived from these $mauq\bar{u}fs$ can reach infinity, ceteris paribus. This makes it possible for temporary waqf to provide perpetual benefit, and it may convince regulators, practitioners, and Islāmic scholars to consider its wider implementation. This paper also suggests how financial technology implementation could make the aforementioned temporary *wāqf* findings a reality. Furthermore, the mathematical representations used in this study may motivate researchers to come out with quantitative models that include temporary *wāqf* and study its impact on real financial and economic settings. The overall aim of this paper is to reemphasize the importance of focusing on the perpetual nature of benefits (substance) rather than on the perpetual nature of $mauq\bar{u}f$ (form). This important shift in perspective can be used to tap into the vast amount of idle assets that could be used to advance social finance and social economy, ensure protection of maslahah, besides ensuring more effective and productive use of existing *waqf* assets.

JEL Classification: C02, D64, A13

Keywords: Revocable *wāqf*, Temporary *wāqf*, *Wāqf muaqqat*, Social finance, Mathematical proof

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1. INTRODUCTION

 $W\bar{a}qf$ or wakaf is commonly known to be perpetual in both its mauque (corpus) and benefit—it cannot be sold, inherited, gifted and revoked. The conventional belief among academicians and practitioners is that *mauquf* should remain permanent in nature and irrevocable in terms of ownership. Another widespread assumption among the masses is that $w\bar{a}qf$ has a strict connotation with pure religious objectives. Although many seminal works in the literature aim at breaking the latter stereotype (Rashid, 2018; Kahf, 2014; Mohsin. 2013; Cizakça, 1998), the former convention about the permanent nature of $mauq\bar{u}f$ is given disproportionately less attention in the literature and therefore to a large extent, remains an unquestioned axiom. One of the possible reasons behind this could be that most of the waqf examples in Muslim history from the very early days were in the form of land, which is a fixed, immovable asset believed to have largely permanent nature. Perhaps one of the most prominent examples is the land of Khaybar, embodied in a hadith based on which one of the key restrictions of wāqf with regard to the mauqūf's permanency or irrevocability was derived:

Ibn Umar reported: Umar acquired a land at Khaibar. He came to Allah's Apostle (may peace be upon him) and sought his advice in regard to it. He said: Allah's Messenger, I have acquired land in Khaibar. I have never acquired property more valuable for me than this, so what do you command me to do with it? Thereupon he (Allah's Apostle) said: If you like, you may keep the corpus intact and give its produce as *şadaqah*. So, Umar gave it as *şadaqah* declaring that property must not be sold or inherited or given away as gift. And Umar devoted it to the poor, to the nearest kin, and to the emancipation of slaves, aired in the way of Allah and guests.... (Muslim)

However, some scholars (Kahf and Mohomed, 2017; Ahmad, 2015) cited multiple instances from the earlier days of Islam when assets that could be considered as temporary in nature, such as horse, weaponries, or cash were indeed used for $w\bar{a}qf$ purposes. Oblivious to most, Umar had also deemed the tools, animals, and slaves, all of which are temporary, as $w\bar{a}qf$ (Kahf, 2014).

Furthermore, the four major *madhabs* (*Islāmic* schools of law) are divided somewhat fifty-fifty in their opinion on the permissibility of temporary $w\bar{a}qf$. Shāfi 'ī and Hanbalī schools of thought hold the opinion that the *mauqūf* should follow the key restriction or principle of perpetuity where it must remain intact, be non-perishable, and not cease easily. This opinion is accepted and respected, but it is also useful to explore the perspective of $w\bar{a}qf$ from the Hanafī and Mālikī madhabs; who are of the opinion that *mauqūf* itself need not necessarily be permanent in nature and can be endowed for a limited time—known in the literature as temporary or revocable $w\bar{a}qf$ (henceforth referred to as temporary $w\bar{a}qf$). This is done to preserve consistency in terminology and reflect that temporality could be due to the physical nature of an asset or limited period of endowment.

Indeed, cash and financial assets have been deemed permissible to be endowed as $w\bar{a}qf$ by $fatw\bar{a}s$ ($Isl\bar{a}mic$ rulings). The $fatw\bar{a}s$ on temporary $w\bar{a}qf$ generally cite that the Ottoman rulers had implemented cash $w\bar{a}qf$ during their heyday (Kahf, 2008). Still, it is important to notice that cash may diminish in value due to inflation while financial assets may also cease due to various contingencies. This then would defy the perpetuity principle of $mauq\bar{u}f$ and might create confusion among the general public. Hence, it is best to harmonize the practice of endowing cash and other previously mentioned items of temporary nature with the $M\bar{a}lik\bar{i}$ perspective on temporary $mauq\bar{u}f$. This harmonization can be further supported if the revocable $w\bar{a}qf$ can be shown to share at least one of the most important characteristics of permanent $w\bar{a}qf$ —the ability to generate perpetual benefits.

In this modern technology age, apart from cash, numerous other assets not conforming to the principle of $mauq\bar{u}f$ perpetuity can still provide significant benefits to society. For example, financial assets, vehicles, technological equipment, and others. Cash as well as returns from financial assets may be used as totally interest-free microfinance loans to the less fortunate (Ibrahim, Amir, and Masron, 2013; Kahf, 1998), or as disaster relief aid. For instance, vehicles can transport poor students from home to school and technological equipment might aid hospitals. Even books and software can be donated as $w\bar{a}qf$ for the purpose of advancing knowledge (Negasi, 2017); for education and research activities are resources with value which will never vanish but only increase with time. As a matter of fact, these assets have potential to generate repeated benefits too.

It is undeniable that the repeated benefits in question are purely theoretical in modern times. Yet, this claim on repeated benefits originated from actual past implementation of $w\bar{a}qf$. This, and further scrutiny on the characteristics of temporary $w\bar{a}qf$, indicate that very few temporary $w\bar{a}qf$ are actually observed at present. Thus, it is crucial to provide evidence that continuous benefits can actually be derived from a temporary $w\bar{a}qf$ in order to lend some support to its real life application. As no study supplied the evidence on this matter through pure mathematical expressions, this paper ambitiously attempts to fill this void.

As such, this paper aims at demonstrating that temporary $w\bar{a}qf$ does not necessarily translate into temporary benefit using mathematical derivation. The main aim of this paper is as such; it is to prove that if a reasonably large number of *mauqūf* are endowed by different founders for the same purpose, for a limited time, but at different periods in time, the total benefit derived from that *mauqūf* may be infinite. This characteristic of perpetual benefit is similar to that of the permanent $w\bar{a}qf$. For the sake of clarification, the purpose in question is the beneficiaries of the $w\bar{a}qf$ or benefits derived from the *mauqūf* such as financial return, employment, education, and poverty alleviation; basically any sorts of welfare allowed by the *Sharī'ah*.

The arguments in this paper are divided into five main sections. After this introduction section, the literature pertaining to temporary $w\bar{a}qf$ is reviewed in Section 2. Section 3 describes the method of derivation and proof while Section 4 constructs the mathematical derivation and proof. An explanation of the mathematical proof using example is also given. Lastly, Section 5 and 6 conclude the paper before providing implications of the study.

2. LITERATURE REVIEW

 $W\bar{a}qf$ played a major role in Muslims economy in the past by serving various objectives from building roads, funding education, providing healthcare support, and sponsoring Hajj. Research has shown that $w\bar{a}qf$ is progressive and can still provide those functions in modern times (Ambrose, Hassan and Hanafi, 2018; Mohsin, 2013; Çizakça, 1998) for its law is mostly derived from *ijtihād* (critical thinking) (Kahf, 2014). These multifaceted applications also serve as a basis for categorizing $w\bar{a}qf$. Obaidullah (2008) and Kahf (2014) categorized $w\bar{a}qf$ into philanthropic, private, and religious purposes. Providing public goods is one example of philanthropic $w\bar{a}qf$ while endowing for one's lineage is essentially private $w\bar{a}qf$. An example of religious $w\bar{a}qf$ is the building of a masjid. Philanthropic $w\bar{a}qf$ is also known as $w\bar{a}qf$ *al-ahlī*, or *wāqf al-aūlād* (Mohsin, 2009). Mahamood (2011) divides *wāqf into wakaf 'ām, wakaf khas, and wakaf musytarak*. These divisions are based on the *wāqf* beneficiaries. *Wakaf 'ām* are founded for the general public benefit and *wakaf khas* is for specific beneficiaries. Meanwhile, *wakaf musytarak* is a hybrid *wāqf* which is a mix of *wakaf 'ām* and *wakaf khas*. On the other hand, Mohsin (2009) defines *wakaf musytarak* as a hybrid of philanthropic *wāqf* and private *wāqf*. Note that *wakaf* is simply how *wāqf* is spelled in Malaysia and other countries such as Indonesia and Brunei.

The literal translation of $w\bar{a}qf$ is to halt, to stop, to avert, or to restrain (Osman et al., 2015; Mohsin, 2009). According to its linguistic origin, $w\bar{a}qf$ means to restrain certain asset in order to derive some benefits from it. However, the technical definition of $w\bar{a}qf$ concept differs among the major schools of *Islāmic* thought. Seman, Kamal, and Hoque (2017) provided a comprehensive summary of technical definitions of $w\bar{a}qf$ according to the four major *madhabs*.

The Shāfi'ī and Hanbalī madhabs share the same definition of waqf which is the confinement of both, a property and its usufruct, in perpetuity for God's cause (Mohsin, 2009). Meanwhile, the Hanafi madhab holds two opinions. The founder of the madhab, Imam Abu Hanifah, defines wāaf as similar to 'ārivah (Rashid, 2018; Mohsin, 2009). ' \bar{A} rivah is simply a loan for use where it involves only the transfer of usufruct and not the property (Ahmad and Hassan, 2006). Thus, Imam Abu Hanifah defines $w\bar{a}qf$ as the confinement of not so much a specific property from the founder but rather its usufruct that is to be channeled for charitable purposes. However, the disciples of Imam Abu Hanifah namely Abu Yusuf and Imam Muhammad bin Hasan have contrary opinions. According to them, wāaf should not be viewed as '*ārivah* but as the confinement of property in perpetuity (Rashid, 2018; Mohsin, 2009). Lastly, the Mālikī madhab holds the same definition as Imam Abu Hanifah whereby wāqf can be viewed as '*ārivah* and thus can be revoked.

Breaking down these technical definitions of $w\bar{a}qf$ a few facts can be surmised. The *Shāfi'ī* and *Hanbalī madhabs*, as well as Abu Yusuf and Imam Muhammad bin Hasan believe that both the usufruct and property must be in perpetuity; while the *Mālikī madhab* and Imam Abu Hanifah hold the opposite belief regarding perpetuity of the endowed property. This is how the terms temporary $w\bar{a}qf$ and permanent $w\bar{a}qf$ came to be. Regardless, it is important to note that the technical definitions also share similarities i.e., detaining the property and dedication of usufruct; the differences are only in the finality of endowment. In Malaysia, the temporary $w\bar{a}qf$ is widely known as *wakaf* muaqqat (limited time $w\bar{a}qf$). However, only two states formally recognize temporary $w\bar{a}qf$ which are Terengganu and the Federal Territories (Noor, 2017). Section 18 of the Wakaf (Terengganu) Enactment 2016 states the following:

- 1. A *waqif* may create a *wakaf irsod*, *wakaf muabbad*, *wakaf muaqqat*, *wakaf musya'* or *wakaf musytarak* on any or all of his property.
- 2. For the purpose of this section
 - a. *"wakaf irsod*" means giving any lands as *wakaf* by the government, *baitulmal*, or any corporations, institutions, organizations or any bodies in accordance to *Sharī'ah*;
 - b. "wakaf muabbad" means giving as wakaf for perpetuity;
 - c. "*wakaf muaqqat*" means giving as *wakaf* for a certain period.
 - d. *"wakaf musya*" means giving as *wakaf* any rights on any joint ownership of asset and are not divided; and
 - e. "*wakaf musytarak*" means combining several *wakaf*s including any *wakaf* that was created through *istibdal* (exchange) and *wakaf* share.

Meanwhile, in Section 2 of the Administration of *Islāmic* Law (Federal Territories) Act 1993, the definition of temporary $w\bar{a}qf$ is embedded in the definition of *wakaf khas*. It is defined as $w\bar{a}qf$ that is permanent or $w\bar{a}qf$ within a fixed period of an asset's capital, which is created for religious or welfare purposes recognized by the *Sharī ah*. The definition continues by clarifying that the income of said capital must be given to persons or purposes that had been delineated in that particular $w\bar{a}qf$.

To avoid misunderstanding between temporary $w\bar{a}qf$, $qar\dot{q}$ alhasan, and sadaqah, it is wise to demarcate the differences. Based on the arguments of the different madhabs and the w $\bar{a}qf$ stipulations discussed before, temporary $w\bar{a}qf$ can be defined as detaining an asset and consigning its usufruct within a restricted period for the sake of welfare recognized by the Sharī ah. On the other hand, sadaqah essentially means charity, which can be further broken down into two types namely mandatory charitable giving and voluntary charitable giving (Awang et al., 2017). Temporary $w\bar{a}qf$ in general, is actually categorized under voluntary charitable giving (Awang et al., 2017). Although it is obvious that in both types of sadaqah the donor will not acquire back the asset being donated (hence the same should apply to $w\bar{a}qf$), scholars have differing opinions regarding $w\bar{a}qf$. This caused the emergence of temporary $w\bar{a}qf$ which was already discussed at length previously. Meanwhile, *qard al-hasan* is defined as a benevolent loan without any form of interest. Unlike temporary *wāqf* and *sadaqah*, it is considered as a financing vehicle instead of charity. Yet, *qard al-hasan* may become part of a *wāqf* model as was elucidated in works such as Kahf (1998) as well as Ibrahim et al. (2013).

Returning to the main point of temporary $w\bar{a}qf$, the major reason temporary $w\bar{a}qf$ is alien in Malaysia is probably because of strict adherence to the $Sh\bar{a}fi$ ' \bar{i} ' concept of $w\bar{a}qf$. Perhaps, realizing that temporary $w\bar{a}qf$ share certain important characteristics with permanent $w\bar{a}qf$, the perception can be changed. Similarly, the Malaysian National Fatwa Council (NFC) has approved some forms of temporary $w\bar{a}qf$. For instance, the NFC has consented that a land on leasehold can be endowed as $w\bar{a}qf$ (Asni and Sulong, 2017). Furthermore, Omar (2018) elucidates those Malaysian regulators have formally recognized temporary $w\bar{a}qf$ to some extent. This, combined with the positive views of some Malaysian '*ulamā*' (*Islāmic* scholars) on temporary $w\bar{a}qf$ show there is a general need for it. This also indicates that there is potential for vast future implementation of temporary $w\bar{a}qf$ in Malaysia.

Another reason temporary $w\bar{a}qf$ is generally ignored by scholars is that in the earlier days of Islam, most of the $w\bar{a}qf$ was in the form of land—the asset which is believed to be perpetual in nature. However, if considered more carefully, even land cannot be assured to be perpetual in nature as it is also subject to erosion and destruction over time or due to calamities, as rightfully pointed out by Chowdhury, Ghazali, and Ibrahim (2011). This had indeed happened in the 13th century when the Dar-al Hikmah library or the Grand Library of Baghdad was annihilated (Mahamood and Rahman, 2015) by the Mongol invader Hulagu Khan. Despite existence of various traditional and innovative modes of $w\bar{a}qf$ financing that can redevelop old $w\bar{a}qf$ properties (Mohsin et al., 2016), none had yet been adopted to regain the main benefits of Dar-al Hikmah library as the epicenter of knowledge, let alone the structure.

Given its potential advantages, it is paradoxical that various reasons were given for excluding temporary $w\bar{a}qf$ assets in the light of the overarching objective of $w\bar{a}qf$ to continuously improve social wellbeing. Even an asset provided as temporary $mauq\bar{u}f$ can generate perpetual benefits. Mohammad, Iman and Hamid (2006) had demonstrated that the $mauq\bar{u}f$ should be viewed separately from the benefits it can provide and the latter should be valued. For example, cash $w\bar{a}qf$ can be given as a microfinance loan totally free of interest (Ibrahim et al., 2013; Kahf, 1998) to a poor family to start their own small business. This cash, despite being temporary $mauq\bar{u}f$, can provide long-lasting repeated benefits, and even potentially perpetual in nature, as this family business might be inherited from generation to generation (for the sake of clarity, it is assumed that cash is a temporary $mauq\bar{u}f$ because (1) one family unit can only enjoy the cash for a short period before repaying the loan and, (2) cash reduces in value over time because of time value of money). From another viewpoint, when applying basic financial valuation principles, the value of the said business might be found to be the same or even surpass the value of the original $mauq\bar{u}f$ (i.e., the original amount of cash endowed), thus still conforming to the principles of perpetuity and inalienability. This surpassed value may be due to the increase in the profit rate.

Indeed, this view of perpetual benefit had ignited a new strand of studies in financial instruments based on temporary $w\bar{a}qf$. Alani (2014) had suggested to combine temporary $w\bar{a}qf$ with modes of *Islāmic* contracts in financing micro projects. Meanwhile, Sulaiman et al. (2019) proposed a unit trust $w\bar{a}qf$ while Jafri and Noor (2019) proposed an *Islāmic* private retirement scheme; both ideas are based on temporary $w\bar{a}qf$. Another team of researchers explored how the permissibility of temporary $w\bar{a}qf$ can open avenues for issuing $w\bar{a}qf$ *sukuk* to finance $w\bar{a}qf$ projects so that a vast amount of idle immovable $w\bar{a}qf$ resources can start generating benefits for society (Negasi, Laeba, and Sani, 2018). Yet, these new strands of studies are only focusing on cash and financial vehicles. Very few studies discuss other forms of *mauqūf* (such as buildings, land, machinery, or equipment etc.) of temporary $w\bar{a}qf$, especially the fact that they may generate perpetual benefits.

Another interesting observation, reiterated by Seman et al. (2017), is that temporary $w\bar{a}qf$ assets are necessary to make the most efficient use of the existing accumulated perpetual $w\bar{a}qf$ assets. For example, machinery and equipment would be necessary in order to cultivate or develop $w\bar{a}qf$ land. This inference is actually similar to Umar's view of tools used in cultivating the land of Khaybar as $w\bar{a}qf$ (elucidated in Section 1). $W\bar{a}qf$ management might have serious difficulties in acquiring financing for this purpose. Chowdhury et al. (2011) highlighted that " $w\bar{a}qf$ land is not valid security and Malaysian banks do not offer credit facility for collateralization of $w\bar{a}qf$ land" (p. 12158). Therefore, a vast amount of $w\bar{a}qf$ land remains underdeveloped and underutilized because of lack of investments (Rashid, 2018; Chowdhury et al., 2011). This is how the asset that

could generate social benefits in perpetuity, remains rather perpetually idle thus falling short of fulfilling the highest and the ultimate objective of the *wāqf* principle.

Rashid (2018) echoes and complements the above argument by reinstating the importance of developing already existing and largely idle permanent $w\bar{a}qf$ assets that have been accumulated in a substantial amount over the centuries following the sole emphasis on perpetual nature of endowment assets. Rashid also warns that developing the existing $w\bar{a}qf$ base is important because we hardly see creation of new $w\bar{a}qf$ these days. This could be attributed to some reasons. First is that affluent Muslims either forget or are reluctant to fulfil this ethical duty. Second, not many Muslims are rich and therefore, even if they want to contribute $w\bar{a}qf$ assets, they might be unable to do so. This is especially true when the perpetuity of the *mauqũf* remains the essential eligible characteristic (Haji-Mohiddin, 2015; Chowdhury et al., 2011).

Temporary $w\bar{a}qf$ could be a solution to the aforementioned challenges. Despite this, very little evidence in the literature can support this claim. Even the literature put forward in this section can be inferred as anachronistic, theoretical, and conceptual. No plausible evidence was given to support temporary $w\bar{a}qf$ as a credible solution because almost none is currently observed. Due to this lack of temporary $w\bar{a}qf$ practice, data cannot be gathered to measure and evaluate its success.

Nonetheless, one can simulate a scenario based on the accepted definitions of permanent and temporary $w\bar{a}qf$ that are translated into mathematical terms. These mathematical terms can then test the outcome of that scenario, i.e., the nature of the derived benefits. The simulation can be done using mathematical expressions as proxies. Using these expressions, one can prove that temporary $w\bar{a}qf$ may provide either temporary or continuous benefits. This research aims at proving the latter. In detail, this research attempts to demonstrate that even when $mauq\bar{u}f$, which is temporary in nature, generates only temporary benefits, perpetual benefits can still be realized. Mathematical logic and derivations unlock such a potential as elaborated in the following sections. To date, this method of providing evidence is one of its kind in the literature on $w\bar{a}qf$.

3. METHODOLOGY

The specific method of mathematical proof used in this study is direct proof or proof by construction. This kind of proof is applicable to the situation when the objective is to prove that a specific occurrence of an object exists. In the context of this paper, the aim is to prove that it is possible to construct a $w\bar{a}qf$ that, despite being composed of temporary $mauq\bar{u}f$ elements, generates perpetual or continuous benefits.

Proof by construction consists of the following two steps (Ferry, 2010). The first step is to assume that a statement, *Y*, is true. The second step is to use *Y* to show that another statement, *Z*, is true. From here, one should show that $Y \Rightarrow Z$ directly. Mathematical expression $Y \Rightarrow Z$ literally denotes the statement "if *Y* is true, then *Z* is true; but if *Y* is false it does not mean that *Z* is false". As such, $Y \Rightarrow Z$ is the proposition that is to be proved. In simple terms, the essence of the proof by construction method is to demonstrate how an object can be built in order to prove that it can exist. Since the aim of this article is to prove that temporary $w\bar{a}qf$ khas, then statement *Y* is "temporary $w\bar{a}qf$ khas is approximately similar to permanent $w\bar{a}qf$ khas can be perpetual".

Yet before proving that $Y \Longrightarrow Z$, several definitions of $w\bar{a}qf$ obtained from the literature review must be delineated to avoid ambiguity in the mathematical proof (Hammack, 2013). Below are the definitions:

- Definition 1.1 A $w\bar{a}qf$ is endowing a $mauq\bar{u}f$ in order to derive some benefits from the said $mauq\bar{u}f$.
- Definition 1.2 A $w\bar{a}qf$ is known as a permanent $w\bar{a}qf$ if and only if a $mauq\bar{u}f$ is endowed for an infinite period (permanently) for the purpose of deriving benefits from the said $mauq\bar{u}f$ perpetually.
- Definition 1.3 A permanent $w\bar{a}qf$ is known as a permanent $w\bar{a}qf$ khas if and only if a certain $mauq\bar{u}f$ is endowed for an infinite period (permanently) for the purpose of deriving a specific benefit from the said $mauq\bar{u}f$ perpetually.
- Definition 1.4 A *wāqf* is known as a temporary *wāqf* if and only if a *mauqūf* is endowed for a finite period (temporarily) for the purpose of deriving benefits from the said *mauqūf* finitely.
- Definition 1.5 A temporary $w\bar{a}qf$ is known as a temporary $w\bar{a}qf$ khas if and only if a certain $mauq\bar{u}f$ is endowed for a finite period (temporarily) for the purpose of deriving a specific benefit from the $mauq\bar{u}f$ finitely.

Definition 1.6 Mauqūf is an asset that is endowed as wāqf.
Definition 1.7 Benefit is the usufruct that a mauqūf provides.
Definition 1.8 Perpetual benefit is the usufruct that a mauqūf provides continuously in perpetuity (permanently).
Definition 1.9 Founder is the person who endows wāqf or the endower of wāaf.

After definitions are put forward, the next step in direct proof is to state a proposition followed by corollary (Hammack, 2013). As was alluded earlier in this section, the proposition is as follows; if temporary $w\bar{a}qf$ khas is approximately similar to permanent $w\bar{a}qf$ khas, then the benefit from temporary $w\bar{a}qf$ khas can be perpetual ($Y \implies Z$).

4. MATHEMATICAL DERIVATION AND PROOF

- Proposition If temporary $w\bar{a}qf$ khas is approximately similar to permanent $w\bar{a}qf$ khas, then the benefit from temporary $w\bar{a}qf$ khas can be perpetual.
- Corollary 1 Following on from the proposition, and by virtue of Definitions 1.1, 1.2, and 1.3, permanent *wāqf khas* is categorized as *wāqf*.
- Corollary 2 Following on from the proposition, and by virtue of Definitions 1.1, 1.4, and 1.5, temporary *wāqf khas* is categorized as *wāqf*.
- Corollary 3 In accordance with Definition 1.1, $w\bar{a}qf$ consists of two main elements namely $mauq\bar{u}f$ and benefit; the same applies to temporary $w\bar{a}qf$ khas and permanent $w\bar{a}qf$ khas.

Suppose temporary $w\bar{a}qf$ khas and permanent $w\bar{a}qf$ khas are both $w\bar{a}qf$. Then following Corollary 3, any $w\bar{a}qf$ fund, W, can be mathematically expressed as a set of $mauq\bar{u}f$ asset(s), m_i , dedicated by the $w\bar{a}qf$ founder(s) that generate corresponding benefits b_i , i = 1..n:

$$W = \{ (m_1, b_1), (m_2, b_2), \dots (m_n, b_n) \}$$
(1.a)

If we think of m_i as a size of this asset, we should expect m_i to remain relatively constant over time—physically it cannot increase but may deteriorate. The relationship between m_i and b_i is elucidated in Corollary 4.

Corollary 4 It is logical to expect that the size of a benefit, b_i , associated with the $mauq\bar{u}f m_i$ in W, should be at least proportionate to the size of $mauq\bar{u}f$ or m_i : $b_i \sim m_i$ throughout time.

To be more specific, we could say that:

$$b_i = m_i \cdot f_i(t) \tag{1.0}$$

(1 h)

where $f_i(t)$ is some function of time. This function of time may be similar to basic valuation models in finance that centres around time value of money.

Let $w\bar{a}qf$ fund W^P be a permanent $w\bar{a}qf$ khas fund. So according to Corollary 1 and Corollary 2; $W^P \in W$. W^P consists of permanent $mauq\bar{u}f$ elements m_i^{PO} , i = 1..n, of similar nature. As such, Corollary 5 can be stated as below:

Corollary 5 The total of m_i^{PO} , i = 1..n in W^P can be represented mathematically as a summation of the permanent *mauqūf* elements.

To put Corollary 5 in mathematical terms:

$$M^{PO} = \sum_{i=1}^{n} m_i^{PO}$$
(2.a)

Based on Definition 1.3, these m_i^{PO} , i = 1..n in W^P are endowed to acquire a specific benefit or objective $O_1 = O_2 = \cdots = O_n = 0$. Consequently, another corollary can be surmised:

Corollary 6 The total of m_i^{PO} , i = 1..n in W^P generates corresponding perpetual benefits b_i^{PO} , i = 1..n that when added up, generates total benefit B^{PO}

Since $W^P \in W$ and by virtue of Definition 1.8 and Corollary 3, B^{PO} can be represented mathematically as the following equation:

$$B^{PO} = \int_{t=0}^{\infty} \sum_{i=1}^{n} m_i^{PO} \cdot f_i(t) dt$$
 (2.b)

Also, keeping in mind that elements m_i^{PO} are constants (for m_i^{PO} are endowed permanently) and do not depend on t, we can further transform the equation (2.b) to the following form:

$$B^{PO} = \int_{t=0}^{\infty} \sum_{i=1}^{n} m_i^{PO} \cdot f_i(t) dt = \sum_{i=1}^{n} m_i^{PO} \int_{t=0}^{\infty} f(t) dt$$

= $M^{PO} \int_{t=0}^{\infty} f(t) dt$ (2.c)

Thus, equations (2) are transformed to the following form (3) which can be considered as mathematical representation of the permanent $w\bar{a}qf$ khas.

$$M^{PO} = \sum_{i=1}^{n} m_i^{PO}$$
(3.a)

$$B^{PO} = M^{PO} \int_{t=0}^{\infty} f(t)dt$$
(3.b)

Now let W^T be a temporary $w\bar{a}qf$ khas fund. So according to Corollary 1 and Corollary 2, $W^T \in W$. We can construct fund W^T in such a way that at any given period of time $[\tau_{i-1}, \tau_i]$, i = 1..n, we have a finite number of founders j, $j = 1..k_{\tau_i}$ who endow mauq $\bar{u}f$ s $m_{j,i}^{TO}$ of the same nature for the same objective $O_1 = O_2 = \cdots =$ $O_{k_{\tau_i}} = O$ but for a finite period $[\tau_{i-1}, \tau_i]$. Hence, two further corollaries can be identified.

- Corollary 7 The total of $m_{j,i}^{TO}$, i = 1..n in W^T can be represented mathematically as a summation of the temporary $mauq\bar{u}f$ elements (M^{TO}) .
- Corollary 8 The total of $m_{j,i}^{TO}$, $i = \overline{1..n}$ in W^T generates corresponding benefits $b_{j,i}^{TO}$, $i = \overline{1..n}$ that when added up, generates total benefits B^{TO} .

Note that in Corollary 8, although we expect the benefits to be rather limited to the period of time $[\tau_{i-1}, \tau_i]$, we should not also exclude a

possibility that benefits $b_{j,i}^{TO}$ might extend beyond the period of time $[\tau_{i-1}, \tau_i]$.

Using Corollary 7, Corollary 8, and Corollary 4 (Corollary 4 is relevant because $W^T \in W$), we can construct the equations for M^{TO} and B^{TO} . Table 1 simplifies the mathematical notations and equations pertaining to temporary *wāqf*. By analysing Table 1, we also realise that within any given period of time $[\tau_{i-1}, \tau_i]$, W^T is equivalent to W^P .

Period of endowment	Total <i>mauqūf</i>	Total benefit
$[au_0, au_1]$	$\sum_{j=1}^{k_{\tau_1}} m_{j,\tau_1}^{TO}$	$\int_{t=\tau_0}^{\tau_1} \sum_{j=1}^{k_{\tau_1}} m_{j,\tau_1}^{TO} f(t) dt$
$[\tau_1,\tau_2]$	$\sum_{j=1}^{k_{\tau_2}} m_{j,\tau_2}^{TO}$	$\int_{t=\tau_1}^{\tau_2} \sum_{j=1}^{k_{\tau_2}} m_{j,\tau_2}^{TO} f(t) dt$
$[\tau_{n-1}, \tau_n]$	$\sum_{j=1}^{k_{\tau_n}} m_{j,\tau_n}^{TO}$	$\int_{t=\tau_{n-1}}^{\tau_n}\sum_{j=1}^{k_{\tau_n}}m_{j,\tau_n}^{TO}f(t)dt$
Total over the period $[\tau_0, \tau_n]$	$M^{TO} = \sum_{i=1}^{n} \sum_{j=1}^{k_{\tau_i}} m_{j,\tau_i}^{TO}$	$B^{TO} = \int_{t=\tau_0}^{\tau_n} \sum_{j=1}^{k_{\tau_n}} m_{j,\tau_n}^{TO} f(t) dt$

TABLE 1Construction of Temporary Wāqf

We can show now that if $n \to \infty$, W^T completely resembles W^P . Allowing n to reach to infinity will result in the following equation:

$$M^{TO} = \lim_{n \to \infty} \left(\sum_{i=1}^{n} \sum_{j=1}^{k_{\tau_i}} m_{j,\tau_i}^{TO} \right) = \sum_{i=1}^{\infty} \sum_{j=1}^{k_{\tau_i}} m_{j,\tau_i}^{TO} = \sum_{j=1}^{\infty} m_j^{TO} \quad (5.a)$$
$$B^{TO} = \lim_{n \to \infty} \left(\int_{t=\tau_0}^{\tau_n} \sum_{j=1}^{k_{\tau_n}} m_{j,\tau_n}^{TO} f(t) dt \right)$$
$$= \int_{t=0}^{\infty} \sum_{j=1}^{\infty} m_j^{TO} f(t) dt \quad (5.b)$$
$$= \sum_{j=1}^{\infty} m_j^{TO} \int_{t=0}^{\infty} f(t) dt = M^{TO} \int_{t=0}^{\infty} f(t) dt$$

The right-hand side of Equations (5) and Equations (3) are similar. Essentially, Equations (5) demonstrates that since the same form of temporary *mauqūfs* are endowed for the same objective by different founders in different period of time until infinity, those temporary *mauqūf* essentially become, as a group, a permanent *mauqūf* which, as a group, provides perpetual benefit. Therefore, benefit from temporary *wāqf khas* can be perpetual. As such, it has been proven that if temporary *wāqf* khas is approximately similar to permanent *wāqf khas*, then the benefit from temporary *wāqf* khas can be perpetual or $Y \implies Z$.

4.1 ILLUSTRATION OF THE MATHEMATICAL DERIVATION AND PROOF

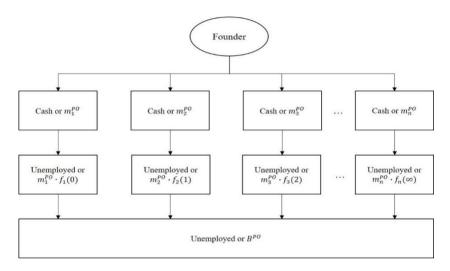
To link the mathematical expressions with example and enhance understanding of the result of the proof, consider this theoretical case of permanent $w\bar{a}qf$ khas in Figure 1. Figure 1 depicts a founder who endows RM10,000 (RM being Malaysian Ringgit) of cash $w\bar{a}qf$ forever. This would generate a permanent $w\bar{a}qf$ khas fund with total mauq $\bar{u}f$ elements of RM10,000 or following Equation 2.a:

$$m_1^{PO} + m_2^{PO} + m_3^{PO} + \dots + m_n^{PO} = \sum_{i=1}^n m_i^{PO} = M^{PO} = \text{RM10,000}$$

The RM10,000 cash $w\bar{a}qf$ is endowed for the purpose of *qard* alhasan to the unemployed to start a business (benefits i.e. $m_1^{PO} \cdot f_1(0)$, $m_2^{PO} \cdot f_2(1)$, $m_3^{PO} \cdot f_3(2)$, ..., $m_n^{PO} \cdot f_n(\infty)$). As such, the RM10,000 of cash *wāqf can* generate permanent benefit to the unemployed as elucidated by Equation 2.b:

$$B^{PO} = m_1^{PO} \cdot f_1(0) + m_2^{PO} \cdot f_2(1) + m_3^{PO} \cdot f_3(2) + \dots + m_n^{PO}$$
$$\cdot f_n(\infty) = \text{RM10,000} \int_{t=0}^{\infty} f(t) dt$$

FIGURE 1 A Theoretical Case of Permanent *Wāqf Khas*



Now to compare, let the theoretical case of cash wāqf demonstrated earlier be a temporary waqf khas. This temporary waqf *khas* is depicted in Figure 2. Assume that there are n number of persons that agree to endow cash (mauq $\bar{u}f$ i.e. $m_{1.1}^{T0}$, $m_{2.2}^{T0}$, $m_{3.3}^{T0}$, ... $m_{\tau_n,n}^{TO}$) as waaf by providing qard al-hasan to the unemployed to start a business (benefits i.e. $m_{1,1}^{TO}f(1), m_{2,2}^{TO}f(2), m_{3,3}^{TO}f(3), \dots, m_{\tau_n,n}^{TO}f(n)$). The founders agree to endow RM10,000 cash in a systematic sequence. Founder 1 agrees to endow RM10,000 for two years (from year 2022 to 2023), Founder 2 endows RM10,000 for three years (from year 2024 to 2026), Founder 3 endows RM10,000 for one year (year 2027 only), and so on until Founder k_{τ_n} . This means that Founder 1 provides RM10,000 in the first period (t = 1) or $[2022_0,$ 2023₁], Founder 2 in the second period (t = 2) or $[2024_1, 2026_2]$, Founder 3 in the third period (t = 3) or $[2027_2, 2027_3]$, and so on until Founder k_{τ_n} in the τ_n th period ($t = \tau_n$) or [τ_{n-1}, τ_n]. Notice in Figure 2 that since this is a temporary wāqf, the cash wāqf is reverted

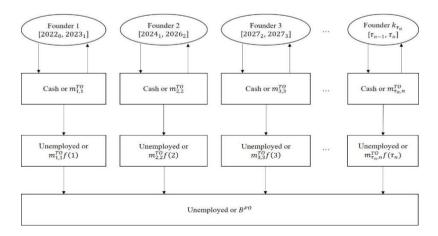
back to the founder after the period of $w\bar{a}qf$ has ended. Notice further that if *n* reaches until infinity, the unemployed as the beneficiary of this temporary $w\bar{a}qf$ khas can receive the benefits of this $w\bar{a}qf$ for long periods of time, approximately the same amount of time akin to a permanent $w\bar{a}qf$ khas illustrated in Figure 1. This is because the RM10,000 of cash $w\bar{a}qf$ is in existence continuously. Placing this example in Equations 5.a and 5.b:

$$\lim_{n \to \infty} (m_{1,1}^{TO} + m_{2,2}^{TO} + \dots + m_{\tau_n,n}^{TO}) = \sum_{i=1}^{\infty} \sum_{j=1}^{\kappa_{\tau_i}} m_{j,\tau_i}^{TO} = \sum_{j=1}^{\infty} m_j^{TO} = M^{TO}$$

= RM10,000
$$B^{TO} = \lim_{n \to \infty} (m_{1,1}^{TO} f(1) + m_{2,2}^{TO} f(2) + m_{3,3}^{TO} f(3) + \dots + m_{\tau_n,n}^{TO} f(\tau_n)) = \lim_{n \to \infty} \left(\int_{t=\tau_0}^{\tau_n} \sum_{j=1}^{\kappa_{\tau_n}} m_{j,\tau_n}^{TO} f(t) dt \right)$$
$$= \int_{t=0}^{\infty} \sum_{j=1}^{\infty} m_j^{TO} f(t) dt = \sum_{j=1}^{\infty} m_j^{TO} \int_{t=0}^{\infty} f(t) dt$$
$$= RM10,000 \int_{t=0}^{\infty} f(t) dt = B^{PO}$$

it can clearly be seen that the benefit from temporary $w\bar{a}qf$ khas can be perpetual.





5. LIMITATION OF STUDY

To the best of our knowledge, no temporary $w\bar{a}qf$ khas in existence has actually demonstrated perpetual benefit. Therefore, the mathematical proof shown in this paper may be deemed as conjecture for it cannot be proven in all possible worlds. In addition, literature review on temporary $w\bar{a}qf$ in the Arabic language were not done for the authors are not well versed in that language. Thus, the literature review may be inadequate.

Regardless, this paper aims at critical reflection on the highly possible nature of perpetual benefits that temporary $w\bar{a}qf$ can produce. This had been demonstrated using the structure from the findings in Section 4. Through the mathematical proof, this paper is able to propose another alternative to safeguard the welfare of $w\bar{a}qf$ beneficiaries continuously i.e., by employing temporary $w\bar{a}qf$ or a type of $w\bar{a}qf$ initially assumed to possess temporary benefit. The result of the direct proof has shed light on the fact that a group of temporary mauq $\bar{u}fs$ may provide perpetual benefit only when the temporary $w\bar{a}qf$ is a wakaf khas and the number of waqifs is large enough. Given the weight of these findings, exclusion of the Arabic language literature on temporary $w\bar{a}qf$ is hopefully compensated.

6. CONCLUSION

This study provides mathematical proof that temporary $w\bar{a}qf$ can provide lasting benefit similar to permanent $w\bar{a}qf$; but only in the case of wakaf khas. This is possible given that an adequate number of founders can endow similar temporary mauq $\bar{u}fs$ for a similar purpose in different periods of time, ceteris paribus. This key finding, that temporary $w\bar{a}qf$ can share essential similarity with permanent $w\bar{a}qf$, greatly contributes to a renewed view of temporary $w\bar{a}qf$ much needed today (Abdullah, 2018). This renewed view is summarized by Kahf (2008, p. 52): " $w\bar{a}qf$ is a perpetual or temporary holding of a mal (asset or usufruct) that produces repeated services, products or revenues for an objective of general or private righteousness."

Temporary $w\bar{a}qf$ model, inspired by the above mathematical derivations, where a large number of founders need to contribute temporary *mauqufs* of a similar nature for the similar purpose, is also in line with the fourth industrial revolution (4IR) which transforms our economy into a largely sharing and increasingly more liquid economy. This is where one can think of utilizing financial technology (fintech) tools such as artificial intelligence (AI), blockchain and Internet of

things (IoT) tools to effectively manage this large number of temporary *mauqūfs* not only in the most transparent way, but also in a clocklike fashion. The blockchain with its smart contract feature (Bogner, Chanson, and Meeuw, 2016) can help to ensure that the temporality of *mauqūfs* is honored. Meanwhile, AI can assist in granting the right *mauqūf* size to the right beneficiary at the right time. This practical fintech implementation is a straight cut for native digital assets such as cash or any other financial assets. With the further advent of 4IR, an increasingly larger and larger amount of physical assets (or rather access to it) is being digitized (Prisco, 2015). This opens a plethora of possibilities for the creative temporary *wāqf* model implementation which can produce unlimited benefits to society.

In conjunction with the derived mathematical proof, this paper may help to persuade Malaysian regulators and practitioners (and other countries that currently shy away from temporary $w\bar{a}qf$) to consider a wider implementation and utilization of temporary $w\bar{a}qf$. In the past, mostly the wealthy endowed $w\bar{a}qf$. With the consideration of temporary $w\bar{a}qf$, the capability, and willingness for everyone to endow increases. As such, the protection of *maslahah* can be better achieved and $w\bar{a}qf$ can penetrate deeper into the fabric of social finance and economics.

Apart from its profound practical implications, this paper adds to the growing literature of *waqf* in several ways by highlighting important gaps and insights and suggesting a host of further research directions. First, research can be done to examine whether temporary wakaf musytarak can provide perpetual benefit. The mathematical derivation in this study had found that temporary wakaf khas can provide perpetual benefits while temporary wakaf 'ām cannot provide such benefits and it is uncertain for the wakaf musytarak case. Second, a qualitative inquiry can be conducted to holistically explore the regulators' and practitioners' willingness to apply temporary waaf with the structure inspired by this paper. Surely, a conceptual paper such as this cannot help the underprivileged if regulators do not allow and are unwilling to implement it. Third, these research findings suggest that temporary wāqf models (including financing) should be developed to solve specific societal needs-the specific similar objective for which temporary *mauqufs* could be continuously endowed. Fourth, a temporary $w\bar{a}qf$ should be included in macroeconomic models to study its impact. Lastly, further research could also focus on case studies of successful temporary wāqf implementation as those can guide researchers and practitioners in temporary *waqf* model development. These initiatives will surely help

 $w\bar{a}qf$ to become an indispensable integral component of the social finance sphere.

Considering the limitations of this study, an experimental design should be adopted in future research so actual implementation of the findings can be conducted and experimented in the real world. By performing this, the mathematical proof can be validated and will not be deemed as conjecture. Future researchers should conduct a systematic literature review (SLR) on temporary $w\bar{a}qf$ by including references from the Arabic language. This method may aid future researchers to accurately gauge the progress of studies in temporary $w\bar{a}qf$.

In conclusion, it is important to once again stress the point that temporality versus perpetuity of $mauq\bar{u}f$ is never the primary focus of $w\bar{a}qf$ concept, as mentioned in texts and manifested in practices of the earlier adherents of Islam. The spirit, the very substance of $w\bar{a}qf$ is to generate repeated benefits for society, while the technicalities of its definition constitute only form. In our time of great liquidity, where borders between temporary and permanent assets are being nearly erased, the substance of $w\bar{a}qf$ needs more careful consideration.

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REFERENCES

- Abdullah, M. "Evolution in Waqf Jurisprudence and Islamic Financial Innovation." *Journal of Islamic Monetary Economics and Finance* 4, no. 1 (2018): 161–82.
- Administration of Islamic Law (Federal Territories) Act 1993. https://tinyurl.com/AdministrationofIslamicLaw
- Alani, U. "Waqf Finance for Micro Projects." Awqaf Journal 27 (2014): 115-62.
- Ambrose, A.H.A.A., M.A.G. Hassan, and H. Hanafi. "A Proposed Model for Waqf Financing Public Goods and Mixed Public Goods in Malaysia." *International Journal of Islamic and Middle Eastern Finance and Management* 11 (2018): 395–415.
- Ahmad, M., and S. Hasan. "Philanthropy and Third Sector in Bangladesh: Overview, Extent, Activities, and Impacts." SSRN,

December 22, 2006.

https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3007055.

___. "Cash Waqf: Historical Evolution, Nature and Role as an Alternative to Riba-Based Financing for the Grass Root." *Journal of Islamic Finance* 176 (2015): 1–12.

- Asni, M.F.A.H.M., and J. Sulong. "Standardisation of Fatwa Regarding Waqf Issues in Malaysia: A Literature Review." *Journal of Fatwa Management and Research* 9, no 1 (2017): 110–28.
- Awang, S.A., F. Muhammad, J.T. Borhan, and M.T. Mohamad. "The Concept of Charity in Islam: An Analysis on the Verses of the Quran and Hadith." *Journal of Usuluddin* 45, no 1 (2017): 141-72.
- Bogner, A., M. Chanson, and A. Meeuw. "A Decentralised Sharing App Running a Smart Contract on the Ethereum Blockchain." In IoT'16: Proceedings of the 6th International Conference on the Internet of Things (2016): 177–8.
- Chowdhury, M.S.R., M.F. Ghazali and M.F. Ibrahim. "Economics of Cash Waqf Management in Malaysia: A Proposed Cash WAQF Model for Practitioners and Future Researchers. *African Journal of Business Management* 5, no. 30 (2011): 12155-63.
- Çizakça, M. "Awqaf in History and its Implications for Modern Islamic Economies." *Islamic Economic Studies* 6 (1998): 43– 70.
- Ferry, David. "Basic Proof Techniques." St. Louis: Washington University in St. Louis, September 13, 2010. https://www.cse.wustl.edu/~cytron/547Pages/f14/IntroToProof s_Final.pdf.
- Haji-Mohiddin, H.A.J.A.H., and N. Mas. "Waqf Development in Malaysia and Singapore: A Comparative Study." Doctoral Dissertation, Durham University, 2015.
- Hammack, Richard. *Book of Proof. Richmond*: Richard Hammack, 2013.
- Ibrahim, H., A. Amir and T.A. Masron. "Cash Waqf: An Innovative Instrument for Economic Development." *International Review* of Social Sciences and Humanities 6 (2013): 1–7.
- Jafri, F.A., and A.M. Noor. "Temporary Waqf Model for Islamic Private Retirement Scheme in Malaysia-A Proposal." *Journal* of Islamic Finance 8, no. 1 (2019): 23–35.
- Kahf, Monzer. Islamic Economics: The Charitable Sector. Qatar: Ad Dawhah, 2014.

__. "Financing the Development of Awqaf Property." *American Journal of Islamic Social Sciences* 16, no. 4 (1999): 39–68.

. "Role of Zakah and Awqaf in Reducing Poverty: A Proposed Institutional Setting within the Spirit of Shari'ah." *Thoughts on Economics* 18 (2008): 40–67.

_____, and A.N. Mohomed. "Cash Waqf: An Innovative Instrument of Personal Finance in Islamic Banking." *Journal of Islamic Economics, Banking, and Finance* 13 (2017): 13–29.

Mahamood, Siti Mashitoh. "Law of Waqf in Malaysia: Recent Developments." In *Essential readings in contemporary waqf issues*, edited by Monzer Kahf and Siti Mashitoh Mahamood, 77–106. Kuala Lumpur: CERT Publications Sdn. Bhd, 2011.

_____, and A. Ab Rahman. "Financing Universities Through Waqf, Pious Endowment: Is it Possible?" *Humanomics* 31 (2015): 430–53.

- Mohammad, M.T.S., M. Iman, and A. Hamid. "Obstacles of the Current Concept of Waqf to the Development of Waqf Properties and the Recommended Alternative." *Malaysian Journal of Real Estate* 1, no. 1 (2006): 27–38.
- Mohsin, Magda Ismail Abdel. *Cash Waqf: A New Financial Product.* Kuala Lumpur: Pearson Malaysia Sdn Bhd, 2009.

_. "Financing through Cash-Waqf: A Revitalization to Finance Different Needs." *International Journal of Islamic and Middle Eastern Finance Management* 6 (2013): 304–21.

- _____, Hisham Dafterdar, Murat Cizakca, Syed Othman Alhabshi, Shaikh Hamzah Abdul Razak, Seyed Kazim Sadr, Thamina Anwar, and Mohammed Obaidullah. *Financing the Development of Old Waqf Properties: Classical Principles and Innovative Practices Around the World*. New York: Palgrave Macmillan, 2016.
- Muslim, al-Hajjāj. "The Book of Wills USC-MSA web (English reference): Book 13, Hadith 4006." sunnah.com, n.d. https://sunnah.com/muslim/25.
- Negasi, M.I. "Financing Higher Education in the Islamic World through Waqf (endowment)." *European Journal of Multidisciplinary Studies* 2, no. 5 (2017): 136–42.

_____, M. Laeba and Y.S. Abubakar. "The Concept of Waqf Sukuk and its Ruling in Islamic Law." *Scholars Journal of Arts, Humanities and Social Sciences* 6, no. 11 (2018): 2073–79.

Noor, Azman Mohd. "Kajian Hukum Wakaf Tunai, Wakaf bertempoh dan Instrumen Kewangan." DrAzman.net, 2017. http://drazman.net/wp-content/uploads/2018/07/Slides_Kajian-Wakaf-tunai-dan-sementara-121217azman.pdf

- Obaidullah, Mohammed. *Introduction to Islamic Microfinance*. India: IBF Net (P) Limited, 2008.
- Omar, Marzunisham. "Waqf An Economic Perspective." Bank for International Settlements, 2018. https://www.bis.org/review/r180411b.html.
- Osman, S., I. Mat, J. Ahmad, and Y. Othman. "Unlocking Value of Waqf Property Using Hibah Mudharabah: A Case Study of Commercial Building in Kedah, Malaysia." *International Journal of Development Research* 5, no. 05 (2015): 4294–99.
- Prisco, Giulio. "Slock.it to Introduce Smart Locks Linked to Smart Ethereum Contracts, Decentralize the Sharing Economy." *Bitcoin Magazine*. November 5, 2015. http://www2.macleans.ca/2011/05/02/spilling-over/
- Rashid, S.K. "Potential of Waqf in Contemporary World." Journal of King Abdulaziz University: Islamic Economics 31, no. 2 (2018): 53-68.
- Seman, A.C., A. Kamal, and M.N. Hoque. "The Development of Principles in the Fiqh of Awqaf: The Principle of Perpetuity Versus Temporality." *Journal of Islamic Economics, Banking and Finance* 13, no. 3 (2017): 30-46.
- Sulaiman, S., A. Hasan, A. M. Noor, M. I. Ismail, and N. H. Noordin. "Proposed Models for Unit Trust Waqf and the Parameters for their Application." *ISRA International Journal of Islamic Finance* 11 (2019): 62-81.
- Wakaf (Terengganu) Enactment 2016 (Jabatan Kehakiman Syariah Terengganu).http://syariah.terengganu.gov.my/files/ENAKME N_WAKAF_2016-WARTA_KERAJAAN.pdf