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TECHNICAL NOTE

LABOUR FORCE ATTACHMENT: EXPLAINING GENDER-DIFFERENCES IN EARNINGS AND EMPLOYMENT

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Abstract

A simple model is constructed to provide an explanation for the observed differences in wages between women and men. Women, if expected to have a weaker attachment to the labour force, will be less accessible to and earn lower wages than men in high-training jobs. This is due to the higher expected turnover costs associated with women. A brief discussion of the predictions of the model and the related policy implications is presented.

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1. Introduction

Over the years, the rate of employment of women has increased and that of men has decreased. Although their labour participation rates appear to be converging, the differences in earnings and occupations between them remain. Full time women workers typically earn about 40 percent less than men.[1] Women also often work in different kinds of jobs. Many female workers are concentrated in labour-intensive occupations such as service jobs which are often characterized by low pay, limited training, and lack of advancement opportunity or job security.[2]

One theory to explain the persistence of differences in earnings and occupation is that there are differences in the expected labour force attachment between women and men. Women are expected to have higher average quit rates because they share a larger proportion of the reasons for quitting. Family responsibilities related to marriage, pregnancy and child-care increase a female's productivity in the household, which, in turn, increases the likelihood of her withdrawing from the labour force. In other words, women have a superior ability in non-market activities which induces a higher probability of separation for women.[3] Studies that present evidence of women having higher average quit rates include Sicherman (1993), Light and Ureta (1992), Barnes and Jones (1974) and Viscusi (1981).

Workers with a weaker attachment to the labour force will have less access to high-wage jobs which typically require substantial training. Turnover is costly in these jobs because any firm-specific training is lost when a worker leaves. Although ability and training are often complementary in production, and workers with higher ability are usually hired into high-training jobs, women of high ability may not be matched into these jobs due to a higher (ex ante) likelihood of them quitting.[4] Landes (1977), in estimating the effects of differences in gender turnover on wages and employment, finds that the relative number of women employed in an occupation varies inversely with the amount of on-the-job training. A similar finding is made by Barron, et al. (1993) who show that women are less likely to fill jobs with a greater duration of training. Both studies suggest that the differences in training may be due to women being perceived as having a weaker attachment to the labour force. However, they do not discount alternative explanations, including discrimination. In another study on training of male and female employees in Britain, Green (1993) finds that there is a considerable degree of discrimination against women. Female workers, with the same personal characteristics as male workers, have a smaller chance of receiving training. Altonji and Spletzer (1991) also demonstrate that although women reportedly have a higher incidence of training, they receive a smaller quantity of training than men.

Differences in earnings across gender will depend on the characteristics of the occupation. The gap between female and male earnings will be greatest in jobs requiring extensive firm-specific training. The wage differential will not be significant in jobs that offer minimal training since turnover rates are not as important when considering wages for these jobs. Landes (1977) finds that training and differences in turnover between women and men explain at least 67 percent of the relative wage differentials within an occupation.

The objective of this paper is to construct a simple model to formalize the issues discussed above. As in Lazear and Rosen (1990), the model demonstrates the relationship between marginal returns to human capital investment (training) and expected labour force participation, and the effect on earnings and employment across gender. Workers are assumed be heterogeneous in their attachment to labour force. The model shows that turnover affects wages in jobs that require training, but not in jobs requiring no training. Consequently, wage differentials will only occur in jobs requiring training, in which workers with a weaker labour force attachment are paid lower wages. The model also assumes that ability affects productivity only in jobs requiring training. It is shown that workers with higher expected turnover rates require a higher ability-level to be matched into training jobs. If women belong to the group of workers with a weaker (expected) attachment to the labour force, the model implies that they will have less access to training jobs; some women will be denied these jobs, which are given, instead, to men with lower ability. For women in jobs requiring training, if the returns to increased ability is higher in the training period than in the later period of employment, they will receive a higher marginal return to ability than men.

2. A Model of Jobs and Wage Determination

Consider a two-period model in which there are two types of workers, M and F. Workers work in period 1, but they work in period 2 only if their wage in that period is greater than the nonmarket alternative value, x, which is a random variable.[5] The distribution function of x for F workers, $G_F(x)$, stochastically dominates the distribution for M workers, $G_M(x)$. That is, $G_F(x) < G_M(x)$ for $x>0$. Workers M and F are assumed to have the same distribution of labour market ability, and each worker's ability, a, is known. Workers can choose either job A or B. Job A entails training in the first period, and a worker in job A produces $\gamma_1(a)$ in the first period, and $\gamma_2(a)$ in the second period; where $\gamma_1(a)<1<\gamma_2(a)$, $\gamma_1'(a)>0$ and $\gamma_2'(a)>0$ for all a. Job B requires no training and a worker produces 1 unit in each period.[6] For simplicity, all workers are paid an identical wage in both periods.[7] It is also assumed that x is uniformly distributed, where,

$$G_F = \int_{\underline{x}_F}^{x} \frac{1}{\bar{x}_F - \underline{x}_F} dz \quad \text{and} \quad G_M = \int_{\underline{x}_M}^{x} \frac{1}{\bar{x}_M - \underline{x}_M} dz; \quad \bar{x}_F > \bar{x}_M > \gamma_2;$$

$\bar{x}_F - \underline{x}_F = \bar{x}_M - \underline{x}_M.$

Competition among firms for workers implies that if a type M worker of ability a is hired for job A, the wage $w_M^A(a)$ will adjust such that profits are zero, or

$$\gamma_1(a) - w_M^A(a) + \frac{w_M^A(a) - \underline{x}_M}{\bar{x}_M - \underline{x}_M}\left(\gamma_2(a) - w_M^A(a)\right) = 0.$$

Similarly, the wage for a type F worker of ability a hired for job A is implicitly defined as:

$$\gamma_1(a) - w_F^A(a) + \frac{w_F^A(a) - \underline{x}_F}{\bar{x}_F - \underline{x}_F}\left(\gamma_2(a) - w_F^A(a)\right) = 0.$$

Type M and F workers hired for job B will be paid their per-period marginal product, i.e.,

(1) $\quad w_M^B(a) = w_F^B(a) = 1.$

In job A, training is given in the first period and the firm reaps the returns in the second period. A worker's likelihood to remain on this job is important to the firm because the returns are lost once the worker leaves. Thus wages paid to workers hired for job A must be adjusted to take into account the cost of turnover; a worker with a higher expected turnover rate will receive lower wages. On the other hand, turnover is irrelevant in job B (a non-training job). The cost to a firm of this worker's departure is zero; a worker with a high turnover rate will receive the same wage as one with a low turnover rate. More formally, we have:

Proposition 1: A type *M* worker with the same ability level as a type *F* worker is paid a higher wage in job A, but there is no difference in wages between type *M* and *F* workers in job B.

(The proof of Proposition 1 and all other propositions are given in the Appendix.)

Since a worker's ability is assumed to affect her productivity (through training) in job A but not (through non-training) in job B, it follows that ability contributes to wages only in job A.

Proposition 2: An increase in ability will increase wages of both type *M* and *F* workers in job A, but ability does not affect wages of either type *M* or *F* workers in job B.

Workers will choose the job which produces the highest expected income. If a type *M* worker of ability a chooses job A, his expected income is:

$$w_M^A(a) + w_M^A(a)\int_{\underline{x}_M}^{w_M^A(a)} \frac{1}{\bar{x}_M - \underline{x}_M}\,dx + \int_{w_M^A(a)}^{\bar{x}_M} \frac{x}{\bar{x}_M - \underline{x}_M}\,dx$$

$$= w_M^A(a) + w_M^A(a)\frac{w_M^A(a)}{\bar{x}_M - \underline{x}_M} + \frac{\left(\bar{x}_M - w_M^A(a)\right)\left(\bar{x}_M + w_M^A(a)\right)}{2\left(\bar{x}_M - \underline{x}_M\right)}$$

$$= \frac{\left(\bar{x}_M + w_M^A(a)\right)^2 - 4\bar{x}_M w_M^A(a)}{2\left(\bar{x}_M - \underline{x}_M\right)}.$$

If job B is chosen, the expected income is:

$$1+\int_{\underline{x}_M}^{1}\frac{1}{\bar{x}_M-\underline{x}_M}dx+\int_{1}^{\bar{x}_M}\frac{x}{\bar{x}_M-\underline{x}_M}dx$$

$$=\frac{(1+\bar{x}_M)^2-4\underline{x}_M}{2(\bar{x}_M-\underline{x}_M)}.$$

Thus an M worker will choose job A if

$$\text{(2)}\qquad \frac{\left(\bar{x}_M+w_M^A(a)\right)^2-4\underline{x}_M w_M^A(a)}{2(\bar{x}_M-\underline{x}_M)}>\frac{(\bar{x}_M+1)^2-4\underline{x}_M}{2(\bar{x}_M-\underline{x}_M)}.$$

Similarly, a type F worker of ability a will choose job A if:

$$\text{(3)}\qquad \frac{\left(\bar{x}_F+w_F^A(a)\right)^2-4\underline{x}_F w_F^A(a)}{2(\bar{x}_F-\underline{x}_F)}>\frac{(\bar{x}_F+1)^2-4\underline{x}_F}{2(\bar{x}_F-\underline{x}_F)}.$$

Define a_M^* and a_F^* such that

$$\text{(4)}\qquad \frac{\left(\bar{x}_M+w_M^A(a_M^*)\right)^2-4\underline{x}_M w_M^A(a_M^*)}{2(\bar{x}_M-\underline{x}_M)}=\frac{(\bar{x}_M+1)^2-4\underline{x}_M}{2(\bar{x}_M-\underline{x}_M)},\ \text{and}$$

$$\text{(5)}\qquad \frac{\left(\bar{x}_F+w_F^A(a_F^*)\right)^2-4\underline{x}_F w_F^A(a_F^*)}{2(\bar{x}_F-\underline{x}_F)}=\frac{(\bar{x}_F+1)^2-4\underline{x}_F}{2(\bar{x}_F-\underline{x}_F)}.$$

The model assumes complementarity between ability and training; the return to training is higher for a more able worker than a less able one. Thus it is expected that a worker with a higher ability level will obtain higher wages in the job that offers training. However, as stated in Proposition 1, a type *F* worker receives a lower wage than a type *M* worker because of the difference in expected quit propensity. The *F* worker must be somewhat more able than an *M* worker for her to gain by choosing job A since the firm sets wages such that it will be compensated for the higher expected loss in training investment. Propositions 3 and 4 state this formally.

Proposition 3: Overall, workers of high ability will choose job A, and those with lower ability will be matched to job B.

Proposition 4: Some type *M* workers will choose job A whereas some type *F* workers of the same or higher ability will not. Type *F* workers in job A will be, on average, of higher ability than *M* workers in job A.[8]

If the distribution of ability is identical for type *M* and *F* workers, then Proposition 4 implies that a higher proportion of type *F* workers will be in type B jobs.

Proposition 5: If $\gamma_1'(a) > \gamma_2'(a)$,[9] then a type *F* worker has a higher return to ability than a type *M* worker in job A.

If the returns to increased ability are higher in the first period than in the second, and since workers receive identical wages in both periods, type *F* workers must receive a higher increment in their wages with an increase in ability to compensate for the larger likelihood of them quitting in the second period.[10] An alternative explanation for the greater return to education for females is that the return to education reflects not only the ability of a worker, but also her quit propensity. In particular, suppose the revelation of ability to employers is achieved by workers at some cost (cost to obtaining the education). If among women there are those who have a high quit propensity, and others with a low quit propensity, then the low quit women would be more likely to seek out job A.[11]

If women are expected to have a weaker (*ex ante*) attachment to the labour force than men, then the model predicts that gender wage differences will occur, especially in occupations that require substantial training. Women will be matched into less favourable jobs compared to men, even with equal abilities. However, those in high-training (thus, more likely high-paying) jobs will have higher returns to ability compared to their male counterparts. This implies that the gender wage gap will decrease as ability level increases. Thus, in jobs that require high ability levels, the difference in work compensation between men and women would be relatively smaller.

3. Discussion and Conclusion

Women are treated differently when, given employment in a particular job, they receive lower wages than men or when, given equal ability, women occupy less favourable jobs than men. These situations can arise if employers perceive women as having a weaker labour force attachment than men. Firms operating in competitive markets will differentiate wages between groups with different turnover rates to compensate for differences in turnover costs.

This essay attempts to examine the impact of the difference between male and female labour force attachment as a contributor to the observed differences in their wages and occupational types. The simple model constructed shows that gender wage differentials do not occur in all types of occupation; rather, they exist in jobs involving training. The presence of specific on-the-job training makes turnover costly to both firm and workers. Women will receive lower wages, but will obtain higher returns to increased ability in training jobs.

The theoretical model presented above provides one explanation for the observed differences. Apart from women being perceived as having a weaker labour force attachment, there could very well be other explanations to account for this difference, such as subjective discrimination.[12] However, this study suggests that wage and occupational differences will occur if employers assume that female workers are more likely than their male counterparts to leave the workforce.

In Malaysia, differences in earnings and occupation between men and women continue to exist. In the agricultural sector, women are largely concentrated in low-skilled, labour-intensive jobs. In the industrial sector, many women workers are in low-paid, semi-skilled, assembly-type production operations. Similarly, in the service sector, women are under-represented in the higher levels of the occupational ladder. In addition, male-female wage differentials continue to exist in the private sector. (Malaysia, 1991).

The Malaysian government has undertaken various steps in promoting labour force participation of women. However, what is more important is to sustain this participation. The government should be more rigorous in encouraging employers to provide support facilities, such as child-care centers and creches, so that women need not have to leave the work force once they have a family. With more sustained participation, women will no longer be perceived as having higher quit rates than men. This may help to achieve a more equitable compensation for both men and women.

Endnotes

1. See Cain (1986) for a summary of studies of ratios of women's earnings to men's earnings.

2. For a survey of empirical evidence on the relative earnings position

and occupations of women and men around the world, see Terrell (1992). For Malaysia, see Malaysia (1991).

3. Lazear and Rosen (1990) adopt this view.

4. The relationship between women's labour force participation and their wages has raised the "chicken or the egg" problem. Do higher quit rates lead to lower wages, or do lower wages lead to higher quit rates? The human capital approach (Mincer and Polachek (1974)) contends that shorter expected work lives result in lower wages as women make fewer investments in human capital than do men, both in school and afterwards, because they anticipate to periodically drop out of the labour force for child care. Studies that support this theory include Shaw and Shapiro (1987), Ragan and Smith (1981) and Sandell and Shapiro (1980). However, others (for example, Corcoran, Duncan and Ponza (1984) and England (1982)) claim that it is not planned separations that lead to lower investment on the job and lower wages, but, rather, low wages that encourage women to quit the labour force when they have children. Still, another study by Blau and Ferber (1991) on students in the senior class in a university in the U.S. finds that planned labour force participation has no effect on expected earnings profile and gender differences in expected earnings have no effect on the number of years women plan to be in the labour market. To adjust for the "chicken-egg" problem, Gronau (1988) and Lewis and Shorten (1991) adopt a simultaneous equation approach. Using hourly earnings and planned labour force separations as the two basic endogenous variables, Gronau finds that plans to quit have no significant effect on job skill intensity in the case of women, which may be indicative of restrictions women face in their choice of jobs. On the other hand, Lewis and Shorten's findings on Australian subjects are consistent with the human capital theory. They find that gender differentials in the labour force attachment and human capital attainment are important factors in determining the gender composition of occupations.

5. For the workers, the nonmarket alternative value may depend on many factors such as their marital status, household income, number of children, etc., during that period, and thus is random.

6. It has been shown that education (a proxy for ability) contributes

differently to one's earning capacity according to the type of occupation. Higher returns to schooling are usually associated with jobs that require high training. Education generates lower returns in jobs that require labour that is simple, menial, repetitive or interchangeable. For example, Boston (1990) finds that additional years of schooling generate lower returns among secondary sector workers than among primary sector workers. (Primary sector workers accumulate more formal on-the-job training than secondary sector workers.) Thus to simplify the model, I assume ability contributes to output only in training jobs.

7. The results of the model hold even if wages are not identical in both periods, as long as the second period wages are less than the marginal output for that period for job A.

8. Since workers M and F are assumed to have the same distribution on ability, and $a_M^* < a_F^*$, therefore the (conditional) mean ability of women hired in job A, $E_F(a|a > a_F^*)$, will be larger than the (conditional) mean ability of men, $E_M(a|a > a_M^*)$.

9. $\gamma_1'(a) > \gamma_2'(a)$ implies return to increased ability is higher in the first period than in the second period. An explanation for this is that ability increases productivity in both periods, and additionally, ability reduces cost of training in the first period.

10. Researchers have found that returns to education differ across groups. For example, Ganderton and Griffin (1993) investigate the rates of return to education of males for major racial or ethnic groups in the United States. They find that whites receive a higher rate of return for an additional year of schooling than Hispanics and blacks, even after controlling for "child-quality" variables. (Child-quality refers to characteristics acquired through the family that influence how much a child learns from a given amount of education.) A similar study on men by Chiswick (1988) finds that groups with higher levels of schooling tend to have higher rates of return to education. This is consistent with the prediction of the model that returns (in terms of wage increase) are higher for a higher ability (i.e., with more education) worker. Gyimah-

Brempong, et al. (1992) findings are also consistent with the prediction of the model - the returns to education for males exceed that for females; however, the marginal returns to (college) education are higher for females than for males.

11 . This relates to the screening hypothesis and signalling theory. For references, see Arrow (1973) and Spence (1973, 1974).

12 . For instance, Becker's theory of subjective (taste) discrimination, where a group of employees are discriminated against because of employers, co-workers, or clients' having a distaste of associating with those in the group.

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APPENDIX

Proof of Proposition 1:

Let w^A be defined such that:

$$\gamma_1(a) - w^A(a) + \frac{w^A - \underline{x}}{\bar{x} - \underline{x}}\left(\gamma_2(a) - w^A\right) = 0$$

$$\Rightarrow (\bar{x} - \underline{x})\left(\gamma_1(a) - w^A\right) + \left(w^A - \underline{x}\right)\left(\gamma_2(a) - w^A\right) = 0.$$

The difference between F and M workers is that $\underline{x}_F > \underline{x}_M$, with $\bar{x}_F - \underline{x}_F = \bar{x}_M - \underline{x}_M$. Thus differentiating totally the above equation with respect to w^A and $\underline{x}$, holding $(\bar{x} - \underline{x})$ constant to obtain:

$$\frac{\partial w^A}{\partial \underline{x}} = \frac{\gamma_2(a) - w^A}{-(\bar{x} - \underline{x}) + \gamma_2(a) + \underline{x} - 2w^A}$$

$$= -\frac{\gamma_2(a) - w^A}{\left(\bar{x} - \gamma_2(a)\underline{x}\right) + 2\left(w^A - \underline{x}\right)} < 0$$

Therefore, $w_M^A(a) > w_F^A(a)$.

From (1), $w_M^B(a) - w_F^B(a) = 0$.

Proof of Proposition 2:

Let w^A be defined such that:

$$(\bar{x} - \underline{x})\left(\gamma_1(a) - w^A\right) + \left(w^A - \underline{x}\right)\left(\gamma_2(a) - w^A\right) = 0.$$

Differentiate totally with respect to w^A and a to obtain:

$$\frac{\partial w^A}{\partial a} = -\frac{(\bar{x}-\underline{x})\gamma_1'(a)+\left(w^A-\underline{x}\right)\gamma_2'(a)}{-(\bar{x}-\underline{x})+\gamma_2(a)+\underline{x}-2w^A} > 0.$$

$$\frac{dw_M^B(a)}{da} = \frac{dw_F^B(a)}{da} = 0.$$

Proof of Proposition 3:

From equations (2), (3), (4) and (5), and Proposition 2, type *M* and *F* workers with ability level higher than a_M^* and a_F^*, respectively, will choose job A.

Proof of Proposition 4:

Let a^* be defined such that

$$\frac{\left(\bar{x}+w^A(a^*)\right)^2-(\bar{x}+1)^2-4\underline{x}\left(w^A(a^*)-1\right)}{2(\bar{x}-\underline{x})} = 0..$$

Differentiate the above equation totally with respect to a^* and $\underline{x}$, and a^* and $\bar{x}$, respectively, to obtain:

$$-\left\{(\bar{x}-\underline{x})\gamma_1'(a)+\left(w^A-\underline{x}\right)\gamma_2'(a)\right\}\frac{\partial a^*}{\partial \underline{x}}$$

$$=\frac{1}{4(\bar{x}-\underline{x})^2}\left\{-8(\bar{x}-\underline{x})\left(w^A(a^*)-1\right)+2\left[\left(\bar{x}+w^A(a^*)\right)^2-(\bar{x}+1)^2-4\underline{x}\left(w^A(a^*)-1\right)\right]\right\};$$

and

$$-\left\{(\bar{x}-\underline{x})\gamma_1'(a)+\left(w^A-\underline{x}\right)\gamma_2'(a)\right\}\frac{\partial a^*}{\partial \bar{x}}$$

$$=\frac{1}{4(\bar{x}-\underline{x})^2}\left\{-4(\bar{x}-\underline{x})\left(w^A(a^*)-1\right)-2\left[\left(\bar{x}+w^A(a^*)\right)^2-(\bar{x}+1)^2-4\underline{x}\left(w^A(a^*)-1\right)\right]\right\};$$

$$\frac{\partial a^*}{\partial \underline{x}}+\frac{\partial a^*}{\partial \bar{x}}=\frac{\left\{-4(\bar{x}-\underline{x})\left(w^A(a^*)-1\right)\right\}}{4(\bar{x}-\underline{x})^2\left((\bar{x}-\underline{x})\gamma_1'(a)+\left(w^A-\underline{x}\right)\gamma_2'(a)\right)}>0.$$

Therefore, $a_M^* < a_F^*$.

Proof of Proposition 5:

From proof of Proposition 2,

$$\frac{\partial w^A}{\partial a}=-\frac{(\bar{x}-\underline{x})\gamma_1'(a)+\left(w^A-\underline{x}\right)\gamma_2'(a)}{-(\bar{x}-\underline{x})+\gamma_2(a)+\underline{x}-2w^A}.$$

Holding $(\bar{x}-\underline{x})$ constant,

$$\frac{\partial^2 w^A}{\partial a\partial \underline{x}}=-\frac{\left[-\gamma_2'(a)\left(-(\bar{x}-\underline{x})+\gamma_2(a)-2w^A+\underline{x}\right)\right]-(\bar{x}-\underline{x})\gamma_1'(a)-\left(w^A-\underline{x}\right)\gamma_2'(a)}{\left[-(\bar{x}-\underline{x})+\gamma_2(a)+\underline{x}-2w^A\right]^2}$$

$$=-\frac{\left[-(\bar{x}-\underline{x})\left(\gamma_1'(a)-\gamma_2'(a)\right)+\gamma_2'(a)\left(\gamma_2(a)-w^A\right)\right]}{\left[-(\bar{x}-\underline{x})+\gamma_2(a)+\underline{x}-2w^A\right]^2}>0.$$

If $\gamma_1'(a)>\gamma_2'(a)$ then $\frac{dw_M^A}{da}<\frac{dw_F^A}{da}$.