International Journal of Economics,Management and Accounting 20, no. 2 (2012): 149-186

ESTIMATION ISSUES AND MATHEMATICAL DERIVATION OF EDUCATIONAL PRODUCTION FUNCTION

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ABSTRACT

A critical assessment of the extant literature of educational production function is discussed in this paper. The discussion covers two important aspects of research development in the area. First, the various approaches used in the estimation of educational production functions, their strengths and weaknesses, are analysed. The main objective of the exercise is to arrive at a shared understanding of the appropriate approach to modelling an educational production function. Second, the general relationship between the input and the output of education is identified from the extant literature. An identification of the relationship is instrumental in terms of variable selection for an empirical study. Once the underlying issues related to the estimation of educational production function are recognised, the derivation of three mathematical models of the function that can be applied in empirical works is provided.

JEL classification: I120, I121.

Keywords: Educational production function, Students' performance, Input-output, Panel data, Endogeneity

1. INTRODUCTION

Education can be considered to be analogous to a production process. The process involves a transformation of educational inputs into educational outputs. The transformation involves teaching and learning processes (education process) that usually takes place in formal institutions, such as schools and universities. The mathematical form of the process is commonly known as an educational production function. It shows the relationship between alternative combinations of educational inputs and educational outputs, given a production technology.

Educational output is typically measured by students' academic performance. Variables such as average test scores, the percentage of an enrolment progressing to the next level of education or the percentage of graduate employment are often employed to represent educational outputs. Educational inputs can be classified into four major factors namely: student/family background characteristics; peer or community influence; school resources and innate abilities.[1] Research to find a statistically robust regression of an educational production function started with Coleman et al., (1966) study.[2] Since then, a considerable amount of research effort has been expended to estimate the parameters of the underlying production functions [see Houtenville and Conway (2008), Mayston (2003), Monk (1989) and Hanushek (1979)]. A few systematic relationships between educational inputs and educational outputs have been confirmed and are discussed in Section 5.

The paper starts with an examination of the various models of the educational production function found in the extant literature. In Section 3, methodological issues that are often raised in the estimation of an educational production function are discussed. Empirical findings from the reviewed literature follow in Section 4. The conclusion of the chapter is presented in Section 5.

2. MODELS OF EDUCATIONAL PRODUCTION FUNCTION

Three empirical models that can be considered central in the extant literature on educational production function are: (i) the contemporaneous education production model, (ii) the value-added model, and (iii) the linear growth (or the gains) model. In this section, the models are analysed and their strengths and weaknesses are compared in order to recognise the appropriate method and dataset required for an empirical work.

A general empirical model of educational production function is presented before the three models are discussed. The general model sets a basic theoretical framework for the estimation of an educational

[3] student academic achievement depends on the combinations of current and past educational inputs. The process can be written as:

$$\text{(1)} \qquad \mathbf{A}_{ijT} = f_T(\mathbf{F}_{ijT} \ldots \mathbf{F}_{ij0},\ \mathbf{P}_{ijT} \ldots \mathbf{P}_{ij0},\ \mathbf{S}_{ijT} \ldots \mathbf{S}_{ij0},\ \mathbf{I}_i)$$

where $\mathbf{A}_{ijT}$ represents a measure of achievement for the i^{th} student at school *j* at time *T;* capital letter *T* denotes the current time; small letter $t = 0$ corresponds to the time interval prior to the time the individual enters school, $t = 1$ corresponds to the first year of school, and $t = 2$ to the second year, and so forth. The notation $\mathbf{F}_{ijT}$ represents a vector of family background influences cumulative to time *T*; $\mathbf{P}_{ijT}$ is a vector of peer (or community) influences cumulative to time *T*; $\mathbf{S}_{ijT}$ is a vector of school inputs cumulative to time *T*; $\mathbf{I}_i$ is a vector of unobserved innate abilities.[4]

Ding and Lehrer (2007)[5] show that equation (1) can be estimated as a linear equation:

$$\text{(2)} \quad \mathbf{A}_{ijT} = \boldsymbol{\beta}_{0T} + \boldsymbol{\beta}_{1T}\mathbf{F}_{ijT} + \boldsymbol{\beta}_{2T}\mathbf{P}_{ijT} + \boldsymbol{\beta}_{3T}\mathbf{S}_{ijT} + \boldsymbol{\beta}_{IT}\mathbf{I}_i + \sum_{t=0}^{T-1}(\boldsymbol{\beta}_{0t} + \boldsymbol{\beta}_{1t}\mathbf{F}_{ijt} + \boldsymbol{\beta}_{2t}\mathbf{P}_{ijt} + \boldsymbol{\beta}_{3t}\mathbf{S}_{ijt}) + \boldsymbol{\varepsilon}_{ijT}$$

In equation (2), betas ($\boldsymbol{\beta}$) represent the parameters to be estimated and $\boldsymbol{\varepsilon}_{ijT}$ is the error term. The independent variables in equation (2) can include higher-order terms and interaction terms to capture non-linear relationships. Lagging equation (2) by one period yields $\mathbf{A}_{ijT-1}$:

$$\text{(3)} \quad \mathbf{A}_{ijT-1} = \boldsymbol{\beta}_{0T-1} + \boldsymbol{\beta}_{1T-1}\mathbf{F}_{ijT-1} + \boldsymbol{\beta}_{2T-1}\mathbf{P}_{ijT-1} + \boldsymbol{\beta}_{3T-1}\mathbf{S}_{ijT-1} + \boldsymbol{\beta}_{IT-1}\mathbf{I}_i + \sum_{t=0}^{T-2}(\boldsymbol{\beta}_{0t} + \boldsymbol{\beta}_{1t}\mathbf{F}_{ijt} + \boldsymbol{\beta}_{2t}\mathbf{P}_{ijt} + \boldsymbol{\beta}_{3t}\mathbf{S}_{ijt}) + \boldsymbol{\varepsilon}_{ijT-1}$$

Equation (3) is important in setting the discussion because some of the terms in the equation will be used in the derivation of the three models of educational production function. Both equations (2) and (3) represent an ideal case when all the input data are available. Researchers, however, resort to one of the three empirical models (as mentioned earlier) depending on the availability of educational input data (Todd & Wolpin, 2003).[6] The application of each of the models to deal with the problem of missing input data is detailed in Appendix 1. Below I describe the underlying theoretical foundation of each of the educational production models.

2.1 CONTEMPORANEOUS EDUCATIONAL PRODUCTION MODEL

The origin of a contemporaneous educational production function can be traced back to Coleman et al. (1966).[7] Hanushek (1986) notes that early studies on education production included only contemporaneous inputs because data on historical inputs were very limited and often not available.

The contemporaneous educational production function requires two central assumptions, namely:

i) Only contemporaneous inputs[8] matter to the production of current achievement. Accordingly, the effect of past educational inputs and unobserved innate ability in the production process decay immediately, or, $\boldsymbol{\beta}_{it} = 0$ for $t = 0, 1, \ldots, T$-1 and $\boldsymbol{\beta}_{IT} = 0$.

ii) Contemporaneous inputs are unrelated to unobserved innate ability and unobserved past educational inputs.

The full derivation of the contemporaneous model from equation (2) is set out in Appendix 1. In essence, the contemporaneous model can be expressed as:

$$\text{(4)} \qquad \mathbf{A}_{ijT} = \boldsymbol{\alpha}_{0T} + \boldsymbol{\alpha}_{1T}\mathbf{F}_{ijT} + \boldsymbol{\alpha}_{2T}\mathbf{P}_{ijT} + \boldsymbol{\alpha}_{3T}\mathbf{S}_{ijT} + \boldsymbol{\varepsilon}^{c}_{ijT}$$

where α's are the parameters to be estimated, and $\boldsymbol{\varepsilon}^{c}_{ijT}$ is the error term. As suggested, the model includes only current measures of educational inputs as explanatory variables.

Unbiased parameter estimates from equation (4) require assumption (i) so that unobserved innate abilities and past inputs to the production process have no effect (such that $\boldsymbol{\beta}_{IT} = 0$ and $\boldsymbol{\beta}_{it} = 0$ for $t = 0, 1, \ldots,$ T-1 on the current achievement.

2.2 VALUE-ADDED EDUCATIONAL PRODUCTION MODEL

The value-added model is the generally acceptable approach among the three models of educational production function (Atkinson, Burgess et al., 2008, and Ding & Lehrer, 2007). The model can be expressed as:

$$\mathbf{A}_{ijT} = \boldsymbol{\gamma}_{0T} + \boldsymbol{\gamma}_{1T}\mathbf{F}_{ijT} + \boldsymbol{\gamma}_{2T}\mathbf{P}_{ijT} + \boldsymbol{\gamma}_{3T}\mathbf{S}_{ijT} + \boldsymbol{\lambda}\mathbf{A}_{ijT-1} + \boldsymbol{\varepsilon}^{L}_{ijT} \tag{5}$$

where γ's and λ's are the parameters to be estimated, and $\boldsymbol{\varepsilon}^{L}_{ijT}$ is the error term.

In essence, equation (5)[9] relates an individual's academic achievement to contemporaneous inputs and a lagged (baseline) achievement measure. The inclusion of previous achievement, $\mathbf{A}_{i\,jT-1}$, in equation (5) is designed to capture the confounding influences of innate abilities, which is often unrecorded due to the lack of data.

Consistent and unbiased parameter estimates of equation (5) require several assumptions to hold (Ding & Lehrer, 2007; Todd & Wolpin, 2003):

i) The effect of observed and unobserved factors in the educational production process should decay over time at the same rate. More specifically, input coefficients must geometrically decline, as measured by time or age, with distance from the achievement measurement (for all *i*), and the rate of decline must be the same for each input). Mathematically, $\boldsymbol{\beta}_{kn} = \boldsymbol{\lambda\beta}_{kn-1}$, where n = 1, 2, …, T and $0 < \boldsymbol{\lambda} < 1$ for k = 0, 1, 2, 3, I. And

ii) $\mathbf{A}_{ijT-1}$ is a sufficient measure of all the previous inputs influences, which includes the unobserved endowment of innate abilities, parental, school and community effects.

There are some considerations that should be taken into account before the model is applied. First, the model places strong restrictions on the production technology when it treats the parameters of innate abilities and past educational inputs as non-age/non-time varying (the effects of the inputs are the same across time, or $\boldsymbol{\beta}_{\kappa T} = \boldsymbol{\beta}_{\kappa T-1} = ... = \boldsymbol{\beta}_{\kappa 0}$ for k = 0, 1, 2, 3, I). Second, data on a lagged measure of achievement is required, in addition to the need for data on contemporaneous family and peer/ community variables that are often lacking.

2.3 LINEAR GROWTH EDUCATIONAL PRODUCTION MODEL

The linear growth model[10] can be dated back to Hanushek (1979). It is expressed as a function of the growth rate in test scores, or mathematically, .

The model is built upon two central assumptions:

i) the unobserved innate ability, $\mathbf{I}_i$, has a constant effect such that, $\boldsymbol{\beta}_{IT} = \boldsymbol{\beta}_{IT-1} = \dots = \boldsymbol{\beta}_{I0} = \varsigma$ where c is a constant,

ii) the test score gain, $\Delta\mathbf{A}_{ijT} = \mathbf{A}_{ijT} - \mathbf{A}_{ijT-1}$, removes the need for data on innate ability, and past educational inputs of family, school and community influences.

Given the assumptions, the linear growth model can be expressed as:

$$\Delta\mathbf{A}_{ijT} = \boldsymbol{\tau}_{0T} + \boldsymbol{\tau}_{1T}\mathbf{F}_{ijT} + \boldsymbol{\tau}_{2T}\mathbf{P}_{ijT} + \boldsymbol{\tau}_{3T}\mathbf{S}_{ijT} + \boldsymbol{\varepsilon}^{G}_{ijT} \tag{6}$$

where τ'sare the parameters of each of the independent variables and $\boldsymbol{\varepsilon}^{G}_{ijT}$ is the error term.

In equation (6), the test score gain is explained by contemporaneous inputs. The unbiased and consistent parameter estimates of equation (6) rely on the assumption that past inputs have a constant effect on achievement at different points in time. Zimmer and Toma (2000, p. 80) suggest that, adding $\mathbf{I}_i$ to equation (6) may improve the estimation results.[11] Since $\mathbf{I}_i$ is one of the important variables that needs to be considered in explaining students' academic achievement, its omission may result in a model misspecification.[12] Notice that the error term, $\boldsymbol{\varepsilon}^{G}_{ijT}$, includes the difference of current and past level of innate abilities.

With the three empirical models of education production in mind, two points merit further consideration. First, notice that all three models rely on strict assumptions. The three models are different in terms of their assumptions about how the impact of observed historical inputs in each production function decay (Todd & Wolpin, 2003) and how each model captures the impact of unobserved innate abilities.[13] Second, in comparison of the three models discussed, the value-added model is most commonly used (Hanushek, 1998). To capture the confounding effects of educational inputs, the value-added model relates an individual's current performance to the individual's performance at some prior time and to the school, community and family inputs during the intervening time.[14] Empirical studies such as Atkinson, Burgess et al. (2008), Koedel (2008), and Houtenville & Conway (2008), favour the value-added model because it provides the most reliable estimates compared to the other two models of educational production function. Equipped with the presentations of the various models of educational production function, methodological issues commonly encountered in research designed to estimate an educational production function are discussed in the next section.

3. METHODOLOGICAL ISSUES

3.1 LEVELS OF ANALYSIS

In the context of estimating an educational production function, the appropriate level of data to be employed is the key to achieve a better understanding of the determinants of students' academic performance. Most studies that estimate educational production function use aggregate data at the district and school levels. The drawback of using school or district level data is that the analysis focuses on the identification of the determinants of school or district educational performance,[15] instead of individual student performance. Most studies use aggregate level data due to serious data limitations on student/family background characteristics and peer background characteristics at the individual level (see Levacic and Vignoles, 2002).

3.2 OMITTED VARIABLES

Many educational production function studies suffer from inadequate measures of students' innate ability,[16] peer effect, school context and processes (teaching methods, teacher quality and school management). Omission of any of these variables may result in biased estimates, particularly when one or more omitted variable is correlated with the included independent variables.[17] One solution to the problem is to employ a panel data. The richness of panel data obviates the need for data that may be difficult to obtain.

3.3 FUNCTIONAL FORM

One area of research that has received less attention in the study of educational production functions is in the identification of the appropriate functional form for the production technology. Most empirical work in the literature assumes a linear or Cobb-Douglas functional form (Figlio, 1997, p. 242).

3.4 ENDOGENEITY

An endogeneity problem is one issue commonly encountered when estimating an educational production function. The endogeneity problem exists when:

i) Any of the independent variables is jointly determined and one of the variables is omitted.[18] As a result, the independent variable is correlated with the error term $(\mathrm{Cov}\,(\varepsilon, \mathbf{X}) \neq 0)$ in a regression. Or

ii) the dependent variable (student academic achievement) influences the independent variables (educational inputs). In other words, the problem occurs when factors that are supposed to affect a particular outcome, depend themselves on that outcome. If, for example, a budget allocation to a school is influenced by the school's performance, then care should be taken when incorporating the education spending variable to capture the school's performance since it is endogenous.[19] Or

iii) past achievement, $\mathbf{A}_{ijT-1}$, is taken as one of the explanatory variables in estimating the educational production function (see Appendix 2).

Without addressing the endogeneity problem, a serious methodological shortcoming arises. The consequence is that estimation results can no longer be interpreted with confidence (Glewwe, 2002, p. 445). The problem can be solved by four main strategies that are elaborated below:

3.4.1 RANDOMISED EXPERIMENTS

One way to eliminate the problem of endogeneity is by having randomised or experimental data. To conduct a randomised experiment, students are assigned to a treatment and a comparison group randomly. The random assignment ensures probabilistic equivalence, where any difference between the treatment and comparison groups is due to chance. The endogeneity problem is eliminated because the randomisation establishes that the intended treatment or program works. In other words, the randomisation provides the assurance (in probability) that the groups are the same before the treatment (program or intervention), and that any difference is due to the treatment. Experimental data, however, is very rare in education. The Tennessee's Student Teacher Achievement Ratio (STAR) project in the US is one of a kind and the scope of the experiment is just on the effect of class size. The experiment involved 11,600 Tennessee kindergarten students and teachers that began in 1985. Krueger (1999) employed the Project STAR data and found a positive effect of small classes on students' academic achievement, particularly for students in the early years of schooling and minority students.

Cook (2007) argues that having this kind of experiment is expensive and may raise ethical issues. Furthermore, Hawthorne effects may prevail since participants may be aware of the experiment and set their behaviour to meet the intended objectives of it (Hoxby, 1998).[20] Krueger (1999) however, notes that the positive effect of smaller classes found in his analysis is free from Hawthorne effects.

3.4.2 SIMULTANEOUS EQUATION MODELS

According to Mayston (1996, p. 131), the number of researchers using simultaneous equation models to estimate an educational production function is small. One difficulty in employing the technique is that a clear understanding of resource allocation process to schools is required. The determinants of school input allocation need to be identified and modelled first. The purpose of that exercise is to make the structural relationships explicit, so that the structural associations between the multiple inputs and outputs of education are untangled (Vignoles et al., 2000). Researchers therefore, need to obtain information on how resources are allocated to schools and this exercise adds another complexity to the task.

3.4.3 INSTRUMENTAL VARIABLES (IV)

The instrumental variable (IV) approach is the more common[21] technique used in the extant literature to deal with the endogenous school resources variable(s). The condition of the IV is that it must be correlated with the endogenous variable and uncorrelated with the error term (the instrument works indirectly through its role as a predictor of the endogenous variable). In the study of school resources based on educational production function, the problem of using the IV approach is the identification of the instruments that influence the allocation of school resources among students but the instruments must not affect the learning outcomes. The IV employed by Figlio (1997), for example, was the tax revenue raising limits that had been imposed in certain US states to identify the random change in educational expenditure.

3.4.4 PANEL DATA APPROACH

A panel data analysis addresses some of the endogeneity by eliminating the effects of unobserved variables. If unobserved traits such as innate ability and motivation are assumed to be time-invariant, then any change in achievement level over time can be regressed on change in school inputs and other observable factors using the Fixed Effects or the Random Effects models. As such, a clean estimate of the effect of the observable factors on students' academic achievement can be achieved since a panel data approach avoids any contamination by students'/ schools' unobserved traits in the regression analysis (Kingdon, 2006, p. 4).

Caution is however, required when employing panel data, particularly in the following cases: (i) when students' unobserved traits change over time. Since panel data models eliminate the effect of unobserved heterogeneity, the change is not accounted for in the model; and (ii) when the cohort of students changes over time due to sample attrition. If students have dropped out of their studies, for example, then the data may comprise only motivated/ambitious/able students (Kingdon, 2006, p. 4).

4. EMPIRICAL EVIDENCE

In this section, findings from the extant literature of educational production function are discussed. The aim is to identify which variables are important in determining educational output. Factors that affect academic achievement, such as family backgrounds, peer influence, school resources and innate abilities, as described in (1), are presented in separate sub-sections.[22]

4.1 FAMILY BACKGROUND AND STUDENT PERFORMANCE

In the literature of educational production function, family background is one important variable found to consistently affect students' academic performance. The effect of positive family backgrounds[23] on a child's academic performance is confirmed in many studies [Houtenville and Conway (2008), McIntosh and Martin (2007), Ammermueller (2007), Rangvid (2007), Woessmann (2004), Henderson & Berla (1994), Nyirongo et al. (1988) and Coleman et al. (1966)]. Okagaki (2001) provides an explanation for this observed phenomenon, stating that a positive family background is usually associated with high familial support.[24]

Factors such as parental education (Burnhill et al., 1990), family wealth (Deon and Pritchett, 2001), and family structure[25] (Pong, 1997 and Krein, 1986) are some of the conventional variables used to analyse the effect of family backgrounds on students' test scores. These variables are considered to fall within the 'home production' side of the educational production function.

Often a set of family variables is used to capture the effect of familial background when estimating an educational production function. Parental education and income are two variables commonly employed

[Rivkin et al. (2005), Goldhaber and Brewer (1996), Ferguson and Ladd (1996), Ehrenberg and Brewer (1994) and Mumane et al. (1981)]. Burnhill et al. (1990), for example, used the number of parents' schooling years and fathers' occupational groups to measure the level of parental education and the level of family incomes, in their estimation of Scotland's educational production function.

Rangvid (2007) employed conventional family variables such as parental education, occupation, wealth and family structure (a student lives with both natural parents) in quantile regressions of Denmark's educational production function. She also included parental academic interest, home educational resources, and cultural possessions in her estimation. Rangvid (2007) argued some of the complex relationships between familial settings and achievement were untangled by adding more family background variables. She found the coefficients on parental education and occupation[26], parental academic interest, educational resources in the home, cultural possessions, ethnicity (being a native Dane) and living with both parents to be positive and significant.

In a study based on Germany's schools, Ammermueller (2007) investigated the determinants of German students' achievement vis-à-vis immigrant students' achievement in PISA examination (in Germany). Ammermueller used dummy variables for parental education and father's unemployment. The other variables that he employed were the number of books at home, number of siblings and language spoken at home. He found significant positive effects of parental education and number of books at home, and negative effects of speaking other than German on students' academic achievement for both categories of students.

McIntosh and Martin (2007) investigated the determinants of educational achievement of Danish students who were 14 years old in 1968, based on the Danish Longitudinal Survey of Youth. Family background variables were found to affect Danish students' achievement. Father's occupation[27] was found to have the largest positive impact on the cohort's achievement. Other variables like parental education[28] (positive), the number of siblings (negative), disrupted childhoods (negative), attitudes towards school (positive), and household income (positive) were also significant.

A recent study by Houtenville and Karen (2008) further confirmed the importance of family background variables.[29] They also found that parental efforts in supporting a child's learning progress were significant and had a strong positive effect on achievement. According to them, parental efforts, however, were not captured by the family background variables employed such as mother's education, father's education (number of years in school), number of siblings, total family income, and percentage of children with a single mother or a single father.[30] Instead, the variable for parental efforts was derived from a ninth-grade student survey that asked how frequently parents; (1) discussed activities or events of particular interest with the child, (2) discussed things the child studied in class, (3) the selection of courses or programs at school, (4) attended a school meeting, and (5) volunteered at the child's school. Note that the data from the survey, in essence, measured the level of familial supports and according to Okagaki (2001), the effectiveness of familial support depends on the parents' level of education and self-efficacy.[31] Since such data on parental efforts, as employed by Houtenville and Karen (2008), are often lacking, parents' level of education remains one variable commonly available to capture the effects of parental effort.

In brief, extant research has confirmed the systematic relationship between family background and students' academic achievement. A set of family background variables that represents parents' education, family income, family structure, and parental effort is crucial to the design of an educational production function. This set of variables captures the main underlying role of familial setting in a child's academic achievement.

4.2 PEER/COMMUNITY INFLUENCE AND STUDENT PERFORMANCE

Peer effect is a change in an individual's behaviour or motivation, caused by the influence of a social group. Researchers separate peer effects into contextual and behavioural effects (Hanushek et al., 2003; Boozer and Cacciola, 2001; Manski, 1993). The contextual effect includes variables that represent group characteristics, such as socio-economic status or race. The behavioural effect refers to a case when an individual outcome is affected by some aspects of the reference group outcomes (for example, achievement of a student may be influenced by a similar achievement of peers). In light of the two effects, analyses on how a student's achievement are affected by the influence of peer academic achievement (Rangvid, 2003; Hanushek et al., 2003), peer race/ethnicity (Ream, 2003), peer socio-economic status and gender (Whitmore, 2005) are common in the literature of educational production function.

To evaluate the contextual effect, the standard practice in the extant literature is to include several peer contextual variables based on race/ethnicity, socio-economic status and gender of peers. Many studies have found a modest negative peer effect based on a racial composition variable[32] (Ream, 2003; Datnow et al., 2003; Bankston and Caldas, 2000). An early study by Hanushek (1972) based on US data found that white students' test scores were negatively affected when the peer group had a very high proportion (greater than 45 percent) of blacks. Further, a study by Angrist and Lang (2004) on a desegregation programme[33] in Boston discovered that mixing black with white students modestly reduced the test scores in the receiving districts.

Another common contextual variable employed is the gender of students. Whitmore (2005) employed the Project STAR data to show the effect of being in a predominantly female class on a student's test score. The independent variable used to capture the gender effect was the proportion of female students in a class room. After disentangling the impact of girls and the impact of higher scoring peers (to separate the effect of induced variations in gender composition and peer quality), Whitmore (2005) found supporting evidence of gender (female) per se on test scores. The estimation result was that having a class predominated by female students had a 1.3 point increase in a student's test score, *ceteris paribus*.[34]

One issue needs consideration when dealing with the contextual variables because the variables tend to be related to parental choice of residential area and/or parental selection of preferred school (for example, public versus private, gender mixed or gender-isolated schools). This selection issue (by parents) may lead to biased estimates.[35] To solve the selection problem, an instrumental variable (IV) for the peer group, assumed to be exogenous, is usually employed. Credible IV that captures the peer effect, however, is difficult to find. Some suggested instrumental variables for the peer group can be based on regional indicators; urbanicity indicators and student body characteristics (Argys, Rees and Brewer, 1996).

With regard to the behavioural peer effect, variables that represent peer quality, such as peer intellectual level[36] and peer behaviour have been employed in the extant literature of educational production function. Hanushek et al. (2003) and Zimmer and Toma (2000) used peer mean test scores to capture the effect of peer intellectual level on students' academic achievement. Zimmer and Toma (2000) reported a robust positive influence of higher achieving peers,[37] where raising the average peer academic level in a group of students could increase an individual student achievement. The findings of Zimmer and Toma (2000) confirmed Summers and Wolfe's (1977) earlier finding that peer effect due to ability grouping was stronger in affecting the achievement level of low-ability students as compared to high-ability students.

In an empirical peer effect study based on Denmark's PISA 2003 data, Rangvid (2003) avoided using peer average test scores as a proxy for peer ability, claiming the potential problem of reverse causality.[38] Instead, Rangvid (2003) employed the average years of schooling of the classmates' mothers, arguing that a large part of a child's academic performance was influenced by the educational level of parents, especially the mother. Rangvid (2003) found strong positive effects of attending school for peers with better educated parents based on OLS results (mean effects). In addition, the quantile regression (median effects) analysis conducted, also showed that peer group effects were stronger at the lower end of the test score distribution. The conclusion based on Rangvid's (2003) quantile regression confirmed further the findings by Zimmer and Toma (2000) and Summers and Wolfe (1977) that low achievers were dependent learners (highly influenced by the achievement of their peers).

Kirk (2000) conducted a behavioural peer effect study based on peer behaviour. He examined the effect of peers' attitude towards their classmates' effort based on the 1998 National Assessment of Educational Progress database of USA for students in fourth and eighth grades. The variable for the peer behaviour was derived from a question in the survey that asked the child to strongly agree, agree, disagree, or strongly disagree with the following statement: "My friends make fun of people who try to do well in school." A negative peer effect was found with a coefficient of 12.26 (negative) in reading test for the fourth grade and 7.003 (negative) for the eighth grade. Kirk (2000) also found that peer behaviour was independent of other factors such as race, gender and income variables.

In summary, the effect of peers on student academic achievement is significant but modest. An empirical analysis to separate peer effects from other confounding influences is econometrically difficult because of the simultaneous nature of peer interactions.[39] Researchers need to identify and obtain data on the salient characteristics of the relevant peer group in order to separate the investigated peer effect from other confounding influences. Any study therefore, must carefully address the endogenous choice of neighbourhood and schools so that the effect of peers on performance can be captured accurately (Moffitt, 2001; Manski 2000).

4.3 SCHOOL RESOURCES AND STUDENT PERFORMANCE

School resources are another important factor that affects students' academic achievement. Class size, teacher quality, and educational expenditure are the common variables investigated under this topic. These variables are discretionary variables of school because they are under a direct control of policymakers (Hanushek, 1986). The main hypothesis under this topic is that greater school resources should have positive effects on students' academic achievement. An evaluation of each of the variables and their impact on students' academic achievement is discussed below.

4.3.1 CLASS SIZE

Evidence from extant literature suggests that class size does affect students' academic achievement in an inverse relationship, especially for students in early years of schooling (Wilby, 2008; Rivkin et al., 2005; Finn et al., 2003; Nye et al., 2002). The range of optimal class size varies across studies from 17 to 25 students per class.[40] Student-teacher ratio and average number of students per class are the variables usually employed to measure the effect of class size on achievement (Rivkin et al., 2005).

The explanation of how class size affects students' academic achievement is explained in Lazear's (2001) analysis of Catholic vis-à-vis public schools in the US. Lazear found an inverse relationship between class size and students' academic achievement. The argument for the inverse relationship was that students in a larger class had less teacher's attention on an individual student and more distraction from other students, resulting in a shorter attention span on a subject being taught as compared to students in a smaller class.

4.3.2 TEACHER QUALITY

With regard to teacher quality, the level of teacher's education (having Master or PhD) and experience are two variables commonly employed. In many studies, teacher quality is found as a major determinant of students' academic achievement (Rivkin et al., 2005; Rice, 2003; Darling-Hammond, 2000. In a review of US studies, Rice (2003) summarised the empirical evidence of teacher quality on students' academic achievement as follows:

i) Teacher experience – Evidence of a positive effect of teacher experience on students' academic achievement was found. The largest effect occurs in the first five years of a teacher's career (1 to 5 years).

ii) Teacher preparation programs and degrees - The prestige of the institution a teacher attended had a positive effect on students' achievement, particularly at the secondary level. Teachers who held advanced degrees had a positive impact on high school students' mathematics and science achievement when the degrees earned were in these subjects.

The significant effects of teacher quality on students' academic achievement, as summarised by Rice (2003) above, was confirmed by several recent studies—see Rivkin et al. (2005), Nye et al. (2004), Wayne and Youngs (2003).

Clotfelter et al. (2006) and Nye et al. (2004) cautioned the potential problem of biased estimates of teacher quality when better trained and more experienced teachers were assigned to teach students of greater ability and with fewer discipline problems. In such a case, there was an upward bias for the estimate of teacher effect because of the positive matching[41] between good students and better teacher quality.[42]

4.3.3 EDUCATIONAL EXPENDITURE

A survey of literature by Hanushek (1989, 1996) involving 187 studies that had been conducted in the US since 1966 concluded that there was no systematic relationship between educational expenditure and student performance. This conclusion sparked heated debates between him and several other researchers who found a positive effect of educational expenditure on students' academic achievement. Ferguson and Ladd (1996) and Hedges et al. (1994a, 1994b), for example, found that educational expenditure was significant in affecting students' academic performance. Hanushek's (1989, 1996) method of aggregating results by counting t-statistics in coming to such a bold conclusion was unsatisfactory, according to his opponents.[43] The appropriate method of combining the results of many studies was not to count the significant t-statistics as Hanushek had done, but to use the tools of meta-analysis, as described by Hedges and Olkin (1980, 1985). Hedges et al. (1994a, 1994b) performed such a meta-analysis and found that taken collectively, the studies surveyed by Hanushek imply the existence of a positive and statistically significant relationship between test scores and expenditure per student.

Jacques and Brorsen (2002) employed school-district data from the Oklahoma Department of Education to investigate the impact of specific categories of expenditure on test scores. They employed 11 expenditure categories[44] as independent variables.[45] They found institutional, student support and transportation expenditures (3 out of 11 expenditure categories) had a statistically significant relationship on students' test scores. The reported magnitude for instructional and transportation expenditures were positive while for student support expenditure was negative.

In summary, the effect of school resources on students' academic achievement is significant. Since school resources are the discretionary variables, results from estimation exercises are important in guiding policymakers to formulate educational policy that aims at improving students' academic achievement.

4.4 INNATE ABILITY AND STUDENT PERFORMANCE

Innate ability is the presence of special talents, attributes or natural aptitudes in an individual. The hypothesis of research on innate ability is that the likelihood of an individual to becoming exceptionally competent in certain fields depends on his/her innate ability.[46] Since innate ability is a non-discretionary variable[47], it is not the main policy variable targeted for an improvement in the achievement of schools (Hanushek, 1986). In estimating an educational production function, however, innate ability is part of an individual student characteristic that should not be omitted, unless a careful model specification adjustment is made (see Appendix 1). Omission of the variable without a careful model specification can cause biased estimates of an educational production function. Hanushek (1986) stated that since innate ability was correlated with a positive family background, then omitting the innate ability variable would cause the estimate of family background to be biased upward.

Data on innate ability, nevertheless, is lacking. The value-added and the linear growth models of educational production function, as explained in Section 2, employ a lagged test score as a sufficient measure for all heritable endowments and historical inputs to overcome the deficiency in data on innate ability, family and community characteristics (Ding and Lehrer, 2007; Woesman, 2006; Todd and Wolpin, 2003; Huang 2002; Hanushek, 1986).[48]

In the value-added model, the implication of employing a lagged test score as an independent variable to represent innate ability can cause an endogeneity problem as I have shown mathematically in Appendix 2. The use of panel data is another way to deal with the problem of data on innate ability.

In brief, estimating the effect of innate ability on performance remains an open area for further research especially in the application of educational production function. Unless data of higher quality is available, estimating the effects of innate ability will remain elusive due to the simplifying assumptions required and the complications of the econometric technical exercise.

5. CONCLUSION

Under the framework of educational production function, educational output, frequently measured in terms of students' academic achievement, is expressed as a function of educational inputs. In doing so, the output depends on past and present educational inputs: (i) student/family background characteristics, (ii) peers/community influences, (iii) school resources, and (iv) innate ability.

From the discussion, this paper found that the estimation of an educational production function is critically susceptible to the problems of data limitation and multi-dimensional interactions of the input-output variables. As a consequence, a pertinent problem of endogeneity must not be ignored. The implication of neglecting the problem is that one ends up with biased estimates of the educational production model. With that in mind, empirical findings from the literature of educational production functions should be interpreted with caution.

It was also found that the availability of data often influenced the type of analysis to be undertaken. The empirical research reviewed varied from a local-level (individual students, schools, or districts level) to a country-level (cross-countries) analysis. Estimations based on the local-level dataset were usually constrained by serious data limitation, particularly the data on student characteristics, family characteristics and peer characteristics. To undertake a country-level analysis, however, data aggregation problems must be considered. In a case of a country-level analysis, data aggregation could result in misleading conclusions regarding the economic behaviour of individuals.

ENDNOTES

1. See papers by Houtenville and Conway (2008), Mayston (2003), Monk (1989) and Hanushek (1979) for further elaboration of the relationship between inputs and outputs in educational production function.

2. The study is known as the Coleman Report. The Report is a national study involving 4,000 public schools in the U.S., which attempts to relate family background (including race and socioeconomic status) and school equity variables (including the integration of white and African-American children) to students' test results and their attitudes towards attending higher education. The Report finds that students' test outcomes are unrelated to the characteristics of schools (for example, the quality of school facilities, programs, and teachers). Instead, the improvement in academic results among minority children is significantly linked to the quality of the family background and students' characteristics—as measured by the proportion of students with encyclopedias in their home and the proportion with high aspirations.

3. See Hanushek (1979) for an early review of conceptual and empirical issues of educational production functions; Vignoles, Levacic et al. (2000) for a more recent review; Todd and Wolpin (2003) for methods on model specification of educational production functions; Meyer and Nascimento (2008) for a review on worldwide findings and methodological issues involving educational production functions.

4. I use the bold type to represent vectors.

5. In their paper, Ding and Lehrer (2007) express the unobservable past inputs with an error term as $\sum_{t=0}^{T-1}(\boldsymbol{\beta}_{0t} + \boldsymbol{\beta}_{1t}\mathbf{F}_{ijt} + \boldsymbol{\beta}_{2t}\mathbf{P}_{ijt} + \boldsymbol{\beta}_{3t}\mathbf{S}_{ijt} + \boldsymbol{\rho}_t\boldsymbol{\varepsilon}_{ijt})$. I exclude the term because the term $\boldsymbol{\varepsilon}_{ijT}$ in equation (2) is assumed to capture all the past unobservable inputs.

6. Input data, especially on family background and innate abilities are rarely available.

7. For an early critical appraisal on the method used in the Coleman Report, see Bowles and Levin (1968). A more current review on model specification of educational production function can be found in Todd and Wolpin (2003).

8. Contemporaneous inputs can be defined as inputs that are close in time to the achievement measure.

9. In Appendix 1, a full derivation of the value-added model from equation (2) is shown.

10. In Appendix 1, a derivation of the linear growth model from equations (2) and (3) is detailed.

11. Zimmer and Toma's (1999) strategy employs data on innate abilities (I) directly, while the original equation (6) implicitly captures the effect of innate abilities in the growth rate of test scores.

12. In the case when one important variable (such as I*i*) is omitted and that variable is correlated with any included explanatory variables (for example parental education), then the effect of I*i* is confounded, resulting in an omitted variable bias.

13. Often data on innate abilities (such as IQ) are not available.

14. See for example the discussion in Hanushek et al., (2003) and Krueger (1999).

15. Data aggregation may result in misleading conclusions regarding the economic behaviour of students.

16. In the case of this type of research, data on individual-specific heterogeneity on student innate ability and motivation are often unobservable.

17. Hanushek (1986), for example, states that since innate ability is correlated with a positive family background, then omitting the innate ability variable will cause the estimate of family background to be biased upward.

18. Resources available to schools are a consequence of factors such as financing rules, school performance and parental choices.

19. Resources available to schools, for example, are a consequence of factors such as student performance, financing rules and parental choices. Parents of high performing students may choose schools which are well equipped. This act of parental choice may result in a positive correlation between performance and school resource.

20. Students perform better just because they are the subject of an experiment, rather than due to the educational intervention itself. Since the experiment may lead to a policy recommendation (smaller classes), the interested parties involved have the incentive for the experiment to work (Hoxby, 1998).

21. As compared to the simultaneous equation approach, data required for the IV approach is less demanding. Researchers only need to employ a suitable instrument under the IV approach. Under the simultaneous equation approach, however, a set of data that explains, for example, how resources are allocated to schools is needed. Such information may not be available.

22. See Fuchs and Wossmann (2007) for international educational production estimates and Alvarez et al. (2007) for state educational production estimates based on Mexico.

23. Positive family backgrounds refer to a conducive physical, mental and emotional environment within a family that stimulate positive child's growth, such as living with both parents, good parental education and income, home library and good parental support and encouragement.

24. Okagaki (2001) suggests that parental involvement influences student achievement via both direct and indirect pathways. The direct pathways involve literal parental engagement with both student homework as well as their involvement in intellectually stimulating activities. This help can be effective depending on the parent's own level of education and self-efficacy. The indirect pathways involve an observation of parental behaviours (positive or negative) by children. A positive/negative spillover effect of the parental behaviours is an outcome of assimilation process by the children of their parental positive/negative behaviours.

25. For example, parental marital status, teen motherhood, single-parent households, and a child's birth order in a family.

26. The reference group contained the unemployed, and those not looking for work. Positive coefficients were found for unskilled and skilled manual workers (0.881 for male students, 0.370 for female students) and for the managerial, professional, and independent or self-employed entrepreneurs (1.258 for male students, 1.029 for female students).

27. Father's occupations were categorised into: (1) the reference group, which contained the unemployed, those not looking for work, and others, (2) unskilled and skilled manual workers, and (3) managerial, professional, and independent or self-employed entrepreneurs.

28. Parental education was categorised into: (1) the reference group, containing no education beyond compulsory schooling, (2) vocational or apprenticeship,

(3) intermediate levels of education leading to white-collar qualifications, and (4) higher levels of education, like universities.

29. The study was based on a value-added educational production function. Data from the National Education Longitudinal Study (NELS) of USA were employed.

30. The implication of Houtenville and Conway (2008) study was that, if parental effort was an important variable and was not captured by the common family background variables, then omitting the parental effort variable could result in biased estimations.

31. Okagaki's (2001) work was reviewed at the beginning of the current section.

32. Racial composition also reflects the socio-economic status of peers when there is a clear economic gap between races.

33. The desegregation programme sends black students from Boston schools to more affluent suburbs (higher numbers of white students).

34. Whitmore (2005) also found that being exposed to higher-quality peers improved a student's test score by 0.6 point for every one point rises in average peer scores.

35. Certain schools are good in ways as observed by parents (for example, some parents may perceive that a gender-isolated school is not good for their child's social development and therefore, send their child to a gender-mixed school) and this unobserved factor may result in biased estimates.

36. Ability grouping is another topic of research that has received great scrutiny. Ability grouping is the practice of dividing students for instruction on the basis of their perceived capacities for learning. Hollifield (1987) argued that ability grouping increased student achievement by reducing the disparity in student ability levels. The advantage of ability grouping was that a teacher could provide instruction based on his students' pace of learning.

37. Zimmer and Toma (2000) conducted a study involving five countries: the US, Belgium, New Zealand, Canada and France.

38. A peer affects his peers and is affected by peers. This interaction may have reverse causality, which may cause a standard regression estimates to

be biased (Rangvid, 2003, p. 16). An instrumental variables (IV) approach is one strategy to solve the problem. The strategy is to use a third variable (instrumental variable) to extract variation in the variable of interest that is unrelated to the causality problem, and to use this variation to estimate its causal effect on an outcome measure.

39. Simultaneous interaction is when a student may affect his/her peers or is affected by peers.

40. For example, in 2003/2004, Alberta's Commission on Learning of Canada had identified 17 to be the ideal number of students for kindergarten, 23 for primary school, 25 for junior high school, and 27 for senior high school classes.

41. Negative matching, on the other hand, could occur when students with low academic achievement were matched with low performing teachers. In such a case, there was a downward bias for the estimate of teacher effect.

42. To overcome the positive matching between better trained and experienced teachers with high ability students, an experimental dataset, formulated based on a random allocation of teachers and students (such as the popular Project STAR data) was one solution.

43. From the 187 previous studies, Hanushek (1989, 1996) counted the percentage of statistically significant evidence of school expenditure variables on student performance with positive and negative signs. Hanushek (1987, 1996) also counted the percentage of statistically insignificant evidence of school expenditure variables on student performance. His conclusion was based on the net outcome between the studies that recorded significant (positive/negative) effects versus the studies that recorded insignificant (positive/negative) effects of school expenditure variables on students' academic achievement.

44. The categories of the expenditures were: (i) instructional expenditures that deal directly with teacher-student interactions, including salaries and benefits for teachers, teacher's aides, clerks, tutors, etc; (ii) instructional support expenditures that assist instructional staff with content and provide tools that enhance the learning process; (iii) student support expenditures on attendance, social work services, guidance services, health services and speech pathology; (iv) school administration expenditures in general supervision of school operations (including staff such as school principals, assistant principals, secretaries and clerks); (v) general administration and

business; (vi) student transportation expenditures; (vii) operations, maintenance, child nutrition, and community service operations; (viii) facilities acquisition and construction expenditures; (ix) other outlays such as debt service, a clearing account, and funds transfer; (x) scholarships; and (xi) repayment.

45. They applied maximum likelihood estimation (MLE) to determine the relationship of the expenditure categories to achievement test scores, controlling for school size, educational attainment of parents, and percentage of students on free/reduced lunch, student race/ethnicity, and proportion of students in special education.

46. Just to cite one influential argument on the presence of innate ability; in one British survey, over three-quarters of the educators who decided which young people were to receive instruction (in music) believed that children could not perform well unless they had special innate gifts (Davis, 1994).

47. Innate ability is not directly amenable to adjustment through economic policy.

48. The models treat pasted individual characteristics as unobservable and invoked assumptions so that the unobservable could be eliminated or ignored (see Appendix 1).

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APPENDIX 1: A DERIVATION OF THE EDUCATIONAL PRODUCTION FUNCTIONS

The purpose of this appendix is to derive the three models of educational production function. Based on the assumptions as outlined in Todd and Wolpin (2003), I use the general forms of the educational production function (equations 2 and 3), as given in Ding and Lehrer (2007), to derive the contemporaneous, value-added and linear growth models. Recall equation (2):

$$\mathbf{A}_{ijT} = \boldsymbol{\beta}_{0T} + \boldsymbol{\beta}_{1T}\mathbf{F}_{ijT} + \boldsymbol{\beta}_{2T}\mathbf{P}_{ijT} + \boldsymbol{\beta}_{3T}\mathbf{S}_{ijT} + \boldsymbol{\beta}_{IT}\mathbf{I}_i + \sum_{t=0}^{T-1}(\boldsymbol{\beta}_{0t} + \boldsymbol{\beta}_{1t}\mathbf{F}_{ijt} + \boldsymbol{\beta}_{2t}\mathbf{P}_{ijt} + \boldsymbol{\beta}_{3t}\mathbf{S}_{ijt}) + \boldsymbol{\varepsilon}_{ijT}$$

and equation (3):

$$\mathbf{A}_{ijT-1} = \boldsymbol{\beta}_{0T-1} + \boldsymbol{\beta}_{1T-1}\mathbf{F}_{ijT-1} + \boldsymbol{\beta}_{2T-1}\mathbf{P}_{ijT-1} + \boldsymbol{\beta}_{3T-1}\mathbf{S}_{ijT-1} + \boldsymbol{\beta}_{IT-1}\mathbf{I}_i + \sum_{t=0}^{T-2}(\boldsymbol{\beta}_{0t} + \boldsymbol{\beta}_{1t}\mathbf{F}_{ijt} + \boldsymbol{\beta}_{2t}\mathbf{P}_{ijt} + \boldsymbol{\beta}_{3t}\mathbf{S}_{ijt}) + \boldsymbol{\varepsilon}_{ijT-1}$$

DERIVING THE CONTEMPORANEOUS EDUCATIONAL PRODUCTION MODEL

Since data on innate ability and past educational inputs are rarely available, the following assumptions are invoked:

i) Only contemporaneous inputs[1] matter to the production of current achievement, in which the effect of past educational inputs and unobserved innate ability in the production process decay immediately, or, $\boldsymbol{\beta}_{it} = 0$ for t = 0, 1, …, T-1 and $\boldsymbol{\beta}_{IT} = 0$.

ii) Contemporaneous inputs are unrelated to unobserved innate ability and unobserved past educational inputs.Assumption (i) eliminates the second terms in equation(2).

Thus, if the assumptions hold, then . $\boldsymbol{\varepsilon}^c_{ijT} = \sum_{t=0}^{T-1}\boldsymbol{\varepsilon}_{ijt}$

However, if the assumptions of the contemporaneous model do not hold, then the unobserved variables in the terms $\boldsymbol{\beta}_{IT}\mathbf{I}_i + \sum_{t=0}^{T-1}(\boldsymbol{\beta}_{0t} + \boldsymbol{\beta}_{1t}\mathbf{F}_{ijt} + \boldsymbol{\beta}_{2t}\mathbf{P}_{ijt} + \boldsymbol{\beta}_{3t}\mathbf{S}_{ijt})$ now appear in the error term component of the following model—the contemporaneous educational production model:

(4) $$\mathbf{A}_{ijT} = \boldsymbol{\alpha}_{0T} + \boldsymbol{\alpha}_{1T}\mathbf{F}_{ijT} + \boldsymbol{\alpha}_{2T}\mathbf{P}_{ijT} + \boldsymbol{\alpha}_{3T}\mathbf{S}_{ijT} + \boldsymbol{\varepsilon}^{c}_{ijT}$$

where $$\boldsymbol{\varepsilon}^{c}_{ijT} = \boldsymbol{\beta}_{IT}\mathbf{I}_i + \sum_{t=0}^{T-1}\left(\boldsymbol{\beta}_{ot} + \boldsymbol{\beta}_{1t}\mathbf{F}_{ijt} + \boldsymbol{\beta}_{2t}\mathbf{P}_{ijt} + \boldsymbol{\beta}_{3t}\mathbf{S}_{ijt}\right) + \boldsymbol{\varepsilon}_{ijT}$$

DERIVING THE VALUE-ADDED EDUCATIONAL PRODUCTION MODEL

In a case when data on innate ability and past educational inputs are not available, let us assume:

i) the effect of observed and unobserved factors in the educational production process should decay over time at the same rate. More specifically, input coefficients must geometrically decline, as measured by time or age, with distance, from the achievement measurement (for all *j*, and the rate of decline must be the same for each input). Mathematically, $\boldsymbol{\beta}_{kn} = \boldsymbol{\lambda}\boldsymbol{\beta}_{kn-1}$, where $n = 1, 2, \ldots, T$ and $0 < \boldsymbol{\lambda} < 1$ for k = 0, 1, 2, 3, I. And

ii) is a sufficient measure of all the previous inputs influences, which includes the unobserved endowment of innate abilities, parental, school and community effects.

Recall equation (2):

$$\mathbf{A}_{ijT} = \boldsymbol{\beta}_{0T} + \boldsymbol{\beta}_{1T}\mathbf{F}_{ijT} + \boldsymbol{\beta}_{2T}\mathbf{P}_{ijT} + \boldsymbol{\beta}_{3T}\mathbf{S}_{ijT} + \boldsymbol{\beta}_{IT}\mathbf{I}_i + \sum_{t=0}^{T-1}(\boldsymbol{\beta}_{0t} + \boldsymbol{\beta}_{1t}\mathbf{F}_{ijt} + \boldsymbol{\beta}_{2t}\mathbf{P}_{ijt} + \boldsymbol{\beta}_{3t}\mathbf{S}_{ijt}) + \boldsymbol{\varepsilon}_{ijT}$$

Lagging equation (2) by one period gives equation (3):

$$\mathbf{A}_{ijT-1} = \boldsymbol{\beta}_{0T-1} + \boldsymbol{\beta}_{1T-1}\mathbf{F}_{ijT-1} + \boldsymbol{\beta}_{2T-1}\mathbf{P}_{ijT-1} + \boldsymbol{\beta}_{3T-1}\mathbf{S}_{ijT-1} + \boldsymbol{\beta}_{IT-1}\mathbf{I}_i + \sum_{t=0}^{T-2}(\boldsymbol{\beta}_{ot} + \boldsymbol{\beta}_{1t}\mathbf{F}_{ijt} + \boldsymbol{\beta}_{2t}\mathbf{P}_{ijt} + \boldsymbol{\beta}_{3t}\mathbf{S}_{ijt}) + \boldsymbol{\varepsilon}_{ijT-1}$$

Multiplying both sides of equation (3) by ‘ yields:

(A1) $$\boldsymbol{\lambda}\mathbf{A}_{ijT-1} = \boldsymbol{\lambda}\boldsymbol{\beta}_{0T-1} + \boldsymbol{\beta}_{1T-1}\boldsymbol{\lambda}\mathbf{F}_{ijT-1} + \boldsymbol{\beta}_{2T-1}\boldsymbol{\lambda}\mathbf{P}_{ijT-1} + \boldsymbol{\beta}_{3T-1}\boldsymbol{\lambda}\mathbf{S}_{ijT-1} + \boldsymbol{\beta}_{IT-1}\boldsymbol{\lambda}\mathbf{I}_i + \sum_{t=0}^{T-2}(\boldsymbol{\lambda}\boldsymbol{\beta}_{0t} + \boldsymbol{\beta}_{1t}\boldsymbol{\lambda}\mathbf{F}_{ijt} + \boldsymbol{\beta}_{2t}\boldsymbol{\lambda}\mathbf{P}_{ijt} + \boldsymbol{\beta}_{3t}\boldsymbol{\lambda}\mathbf{S}_{ijt} +) + \boldsymbol{\lambda}\boldsymbol{\varepsilon}_{ijT-1}$$

Subtracting (A1) from equation (2) gives:

$$\text{(A2)}\quad \begin{aligned}\mathbf{A}_{ijT}-\boldsymbol{\lambda}\mathbf{A}_{ijT-1}=&\left(\boldsymbol{\beta}_{0T}-\boldsymbol{\lambda}\boldsymbol{\beta}_{0T-1}\right)+\left(\boldsymbol{\beta}_{1T}-\boldsymbol{\lambda}\boldsymbol{\beta}_{1T-1}\right)\mathbf{F}_{ijT}+\left(\boldsymbol{\beta}_{2T}-\boldsymbol{\lambda}\boldsymbol{\beta}_{2T-1}\right)\mathbf{P}_{ijT}+\\&\left(\boldsymbol{\beta}_{3T}-\boldsymbol{\lambda}\boldsymbol{\beta}_{3T-1}\right)\mathbf{S}_{ijT}+\left(\boldsymbol{\beta}_{IT}-\boldsymbol{\lambda}\boldsymbol{\beta}_{IT-1}\right)\mathbf{I}_{i}+\\&\sum_{t=0}^{T-1}\left[\left(\boldsymbol{\beta}_{0t}-\boldsymbol{\lambda}\boldsymbol{\beta}_{0t}\right)+\left(\boldsymbol{\beta}_{1t}-\boldsymbol{\lambda}\boldsymbol{\beta}_{it}\right)\mathbf{F}_{ijt}+\left(\boldsymbol{\beta}_{2t}-\boldsymbol{\lambda}\boldsymbol{\beta}_{2t}\right)\mathbf{P}_{ijt}+\left(\boldsymbol{\beta}_{3t}-\boldsymbol{\lambda}\boldsymbol{\beta}_{3t}\right)\mathbf{S}_{ijt}\right]+\boldsymbol{\varepsilon}\end{aligned}$$

Taking the term $\boldsymbol{\lambda}\mathbf{A}_{ijT-1}$ to the right-hand side of equation (A2) yields:

$$\text{(A3)}\quad \begin{aligned}\mathbf{A}_{ijT}=&\left(\boldsymbol{\beta}_{0T}-\boldsymbol{\lambda}\boldsymbol{\beta}_{0T-1}\right)+\left(\boldsymbol{\beta}_{1T}-\boldsymbol{\lambda}\boldsymbol{\beta}_{1T-1}\right)\mathbf{F}_{ijT}+\left(\boldsymbol{\beta}_{2T}-\boldsymbol{\lambda}\boldsymbol{\beta}_{2T-1}\right)\mathbf{P}_{ijT}+\\&\left(\boldsymbol{\beta}_{3T}-\boldsymbol{\lambda}\boldsymbol{\beta}_{3T-1}\right)\mathbf{S}_{ijT}+\boldsymbol{\lambda}\mathbf{A}_{ijT-1}+\left(\boldsymbol{\beta}_{IT}-\boldsymbol{\lambda}\boldsymbol{\beta}_{IT-1}\right)\mathbf{I}_{i}+\\&\sum_{t=0}^{T-1}\left[\left(\boldsymbol{\beta}_{0t}-\boldsymbol{\lambda}\boldsymbol{\beta}_{0t}\right)+\left(\boldsymbol{\beta}_{1t}-\boldsymbol{\lambda}\boldsymbol{\beta}_{it}\right)\mathbf{F}_{ijt}+\left(\boldsymbol{\beta}_{2t}-\boldsymbol{\lambda}\boldsymbol{\beta}_{2t}\right)\mathbf{P}_{ijt}+\left(\boldsymbol{\beta}_{3t}-\boldsymbol{\lambda}\boldsymbol{\beta}_{3t}\right)\mathbf{S}_{ijt}\right]+\boldsymbol{\varepsilon}_{ijT}\end{aligned}$$

And gathering some terms yields:

$$\text{(A4)}\quad \begin{aligned}\mathbf{A}_{ijT}=&\left(\boldsymbol{\beta}_{0T}-\boldsymbol{\lambda}\boldsymbol{\beta}_{0T-1}\right)+\left(\boldsymbol{\beta}_{1T}-\boldsymbol{\lambda}\boldsymbol{\beta}_{1T-1}\right)\mathbf{F}_{ijT}+\left(\boldsymbol{\beta}_{2T}-\boldsymbol{\lambda}\boldsymbol{\beta}_{2T-1}\right)\mathbf{P}_{ijT}+\\&\left(\boldsymbol{\beta}_{3T}-\boldsymbol{\lambda}\boldsymbol{\beta}_{3T-1}\right)\mathbf{S}_{ijT}+\boldsymbol{\lambda}\mathbf{A}_{ijT-1}+\left(\boldsymbol{\beta}_{IT}-\boldsymbol{\lambda}\boldsymbol{\beta}_{IT-1}\right)\mathbf{I}_{i}+\\&\sum_{t=0}^{T-1}\left[(1-\boldsymbol{\lambda})\boldsymbol{\beta}_{0t}+(1-\boldsymbol{\lambda})\boldsymbol{\beta}_{1t}\mathbf{F}_{ijt}+(1-\boldsymbol{\lambda})\boldsymbol{\beta}_{2t}\mathbf{P}_{ijt}+(1-\boldsymbol{\lambda})\boldsymbol{\beta}_{3t}\mathbf{S}_{ijt}\right]+\boldsymbol{\varepsilon}_{ijT}\end{aligned}$$

Re-expressing equation (A4) gives the value-added educational production model:

$$\text{(5)}\quad \mathbf{A}_{ijT}=\boldsymbol{\gamma}_{0T}+\boldsymbol{\gamma}_{1T}\mathbf{F}_{ijT}+\boldsymbol{\gamma}_{2T}\mathbf{P}_{ijT}+\boldsymbol{\gamma}_{3T}\mathbf{S}_{ijT}+\boldsymbol{\lambda}\mathbf{A}_{ijT-1}+\boldsymbol{\varepsilon}_{ijT}^{L}$$

where if the assumptions of the model hold, then $\boldsymbol{\varepsilon}_{ijT}^{L}=\boldsymbol{\varepsilon}_{ijT}$. However, if the assumptions do not hold, then the terms

$$\left(\boldsymbol{\beta}_{IT}-\boldsymbol{\lambda}\boldsymbol{\beta}_{IT}\right)\mathbf{I}_{i}+\sum_{t=0}^{T-1}\left[(1-\boldsymbol{\lambda})\boldsymbol{\beta}_{ot}+(1-\boldsymbol{\lambda})\boldsymbol{\beta}_{1t}\mathbf{F}_{ijt}+(1-\boldsymbol{\lambda})\boldsymbol{\beta}_{2t}\mathbf{P}_{ijt}+(1-\boldsymbol{\lambda})\boldsymbol{\beta}_{3t}\mathbf{S}_{ijt}\right]$$

now appear in the error term component of the model, such that:

$$\boldsymbol{\varepsilon}_{ijT}^{L}=\left(\boldsymbol{\beta}_{IT}-\boldsymbol{\lambda}\boldsymbol{\beta}_{IT}\right)\mathbf{I}_{i}+\sum_{t=0}^{T-1}\left[(1-\boldsymbol{\lambda})\boldsymbol{\beta}_{ot}+(1-\boldsymbol{\lambda})\boldsymbol{\beta}_{1t}\mathbf{F}_{ijt}+(1-\boldsymbol{\lambda})\boldsymbol{\beta}_{2t}\mathbf{P}_{ijt}+(1-\boldsymbol{\lambda})\boldsymbol{\beta}_{3t}\mathbf{S}_{ijt}\right]+\boldsymbol{\varepsilon}_{ijT}$$

DERIVING THE LINEAR GROWTH EDUCATIONAL PRODUCTION MODEL

The following assumptions are made when data on innate ability and past educational inputs are not available.

i) The unobserved innate ability, $\mathbf{I}_i$, has a constant effect such that, $\boldsymbol{\beta}_{IT} = \boldsymbol{\beta}_{IT-1} = ... = \boldsymbol{\beta}_{I0} = c$, where c is a constant.

ii) The test score gain, $\Delta\mathbf{A}_{ijT} = \mathbf{A}_{ijT} - \mathbf{A}_{ijT-1}$, removes the need for data on innate ability, and past educational inputs of family, school and community influences.

Given the assumptions, the linear growth model is derived by:

= [Equation (2)] – [Equation (3)]

$$\Delta\mathbf{A}_{ijT} = \left[\boldsymbol{\beta}_{0T} + \boldsymbol{\beta}_{1T}\mathbf{F}_{ijT} + \boldsymbol{\beta}_{2T}\mathbf{P}_{ijT} + \boldsymbol{\beta}_{3T}\mathbf{S}_{ijT} + \boldsymbol{\beta}_{IT}\mathbf{I}_i + \sum_{t=0}^{T-1}(\boldsymbol{\beta}_{0t} + \boldsymbol{\beta}_{1t}\mathbf{F}_{ijt} + \boldsymbol{\beta}_{2t}\mathbf{P}_{ijt} + \boldsymbol{\beta}_{3t}\mathbf{S}_{ijt}) + \boldsymbol{\varepsilon}_{ijT}\right] -$$
$$\left[\boldsymbol{\beta}_{0T-1} + \boldsymbol{\beta}_{1T-1}\mathbf{F}_{ijT-1} + \boldsymbol{\beta}_{2T-1}\mathbf{P}_{ijT-1} + \boldsymbol{\beta}_{3T-1}\mathbf{S}_{ijT-1} + \boldsymbol{\beta}_{IT-1}\mathbf{I}_i + \sum_{t=0}^{T-2}(\boldsymbol{\beta}_{0t} + \boldsymbol{\beta}_{1t}\mathbf{F}_{ijt} + \boldsymbol{\beta}_{2t}\mathbf{P}_{ijt} + \boldsymbol{\beta}_{3t}\mathbf{S}_{ijt}) + \boldsymbol{\varepsilon}_{ijT-1}\right]$$

Simplifying the above equation yields:

$$\Delta\mathbf{A}_{ijT} = (\boldsymbol{\beta}_{0T} - \boldsymbol{\beta}_{0T-1}) + (\boldsymbol{\beta}_{1T} - \boldsymbol{\beta}_{1T-1})\mathbf{F}_{ijT} + (\boldsymbol{\beta}_{2T} - \boldsymbol{\beta}_{2T-1})\mathbf{P}_{ijT} + (\boldsymbol{\beta}_{3T}\boldsymbol{\beta}_{3T-1})\mathbf{S}_{ijT} + (\boldsymbol{\beta}_{IT} - \boldsymbol{\beta}_{IT-1})\mathbf{I}_i + \boldsymbol{\varepsilon}_{ijT}$$

Given assumption (i) that $\boldsymbol{\beta}_{IT} = \boldsymbol{\beta}_{IT-1} = \boldsymbol{\beta}_{IT-2} = ... = \boldsymbol{\beta}_{IT-m} = c$ then,

$$\Delta\mathbf{A}_{ijT} = (\boldsymbol{\beta}_{0T} - \boldsymbol{\beta}_{0T-1}) + (\boldsymbol{\beta}_{1T} - \boldsymbol{\beta}_{1T-1})\mathbf{F}_{ijT} + (\boldsymbol{\beta}_{2T} - \boldsymbol{\beta}_{2T-1})\mathbf{P}_{ijT} + (\boldsymbol{\beta}_{3T}\boldsymbol{\beta}_{3T-1})\mathbf{S}_{ijT} + \boldsymbol{\varepsilon}_{ijT}$$

Re-expressing the above equation, yields—the linear growth educational production model:

(6) $$\Delta\mathbf{A}_{ijT} = \boldsymbol{\tau}_{0T} + \boldsymbol{\tau}_{1T}\mathbf{F}_{ijT} + \boldsymbol{\tau}_{2T}\mathbf{P}_{ijT} + \boldsymbol{\tau}_{3T}\mathbf{S}_{ijT} + \boldsymbol{\varepsilon}^G_{ijT}$$

where if the assumptions of the model hold, then $\boldsymbol{\varepsilon}^G_{ijT} = \boldsymbol{\varepsilon}_{ijT}$.

However, if the assumptions do not hold, then the error term is given by $\boldsymbol{\varepsilon}^G_{ijT} = \boldsymbol{\varepsilon}_{ijT} + (\boldsymbol{\beta}_{IT} - \boldsymbol{\beta}_{IT-1})\mathbf{I}_i$.

APPENDIX 2:
ENDOGENEITY PROBLEM WHEN A LAGGED TEST SCORE IS USED AS ONE OF THE INDEPENDENT VARIABLES

The purpose of this appendix is to prove the existence of an endogeneity problem when a lagged test score is used as one of the independent variables in the value-added educational production model. I prove the problem based on a manipulation of equations (2) and (3). A simplification is made to equations (2) and (3) for notational convenience. The intercept and all the educational inputs, except **I,** are replaced by $\mathbf{X}_{ijt}$, such that $\mathbf{X}_{ijt} = \left[1\ \mathbf{F}_{ijt}\ \ \mathbf{P}_{ijt}\ \ \mathbf{S}_{ijt}\ \right]$, for t = 0, 1, 2, 3, …, T.

Given the simplification, equation (2) can now be expressed as:

(A5) $$\mathbf{A}_{ijT} = \boldsymbol{\beta}_T \mathbf{X}_{ijT} + \boldsymbol{\beta}_{IT}\mathbf{I}_i + \sum_{t=0}^{T-1}\left(\boldsymbol{\beta}_t \mathbf{X}_{ijt}\right) + \boldsymbol{\varepsilon}_{ijT}$$

while equation (3) can be written as:

(A6) $$\mathbf{A}_{ijT-1} = \boldsymbol{\beta}_{T-1}\mathbf{X}_{ijT-1} + \boldsymbol{\beta}_{IT-1}\mathbf{I}_i + \sum_{t=0}^{T-2}\left(\boldsymbol{\beta}_t \mathbf{X}_{ijt}\right) + \boldsymbol{\varepsilon}_{ijT-1}$$

From equation (A6), re-expressing the equation in terms of unobserved innate abilities yields:

(A7) $$\mathbf{I}_i = \frac{1}{\boldsymbol{\beta}_{IT-1}}\left[\mathbf{A}_{ijT-1} - \boldsymbol{\beta}_{T-1}\mathbf{X}_{ijT-1} - \sum_{t=0}^{T-2}\left(\boldsymbol{\beta}_t \mathbf{X}_{ijt}\right) - \boldsymbol{\varepsilon}_{ijT-1}\right]$$

Substituting equation (A7) into equation (A5) gives:

(A8) $$\mathbf{A}_{ijT} = \boldsymbol{\beta}_T \mathbf{X}_{ijT} + \frac{\boldsymbol{\beta}_{IT}}{\boldsymbol{\beta}_{IT-1}}\left[\mathbf{A}_{ijT-1} - \boldsymbol{\beta}_{T-1}\mathbf{X}_{ijT-1} - \sum_{t=0}^{T-2}\left(\boldsymbol{\beta}_t \mathbf{X}_{ijt}\right) - \boldsymbol{\varepsilon}_{ijT-1}\right] + \sum_{t=0}^{T-1}\left(\boldsymbol{\beta}_t \mathbf{X}_{ijt}\right) + \boldsymbol{\varepsilon}_{ijT}$$

Expanding equation (A8) yields:

$$\text{(A9)}\quad \mathbf{A}_{ijT} = \boldsymbol{\beta}_T \mathbf{X}_{ijT} + \frac{\boldsymbol{\beta}_{IT}}{\boldsymbol{\beta}_{IT-1}} \mathbf{A}_{ijT-1} - \frac{\boldsymbol{\beta}_{IT}}{\boldsymbol{\beta}_{IT-1}} \boldsymbol{\beta}_{T-1} \mathbf{X}_{ijT-1} - \frac{\boldsymbol{\beta}_{IT}}{\boldsymbol{\beta}_{IT-1}} \sum_{t=0}^{T-2} \left(\boldsymbol{\beta}_t \mathbf{X}_{ijt} \right) - \frac{\boldsymbol{\beta}_{IT}}{\boldsymbol{\beta}_{IT-1}} \boldsymbol{\varepsilon}_{ijT-1} + \sum_{t=0}^{T-1} \left(\boldsymbol{\beta}_t \mathbf{X}_{ijt} \right) + \boldsymbol{\varepsilon}_{ijT}$$

In equation (A9), let (A10) be $\boldsymbol{\upsilon}_{i\,jT} = -\frac{\boldsymbol{\beta}_{IT}}{\boldsymbol{\beta}_{IT-1}} \left[\boldsymbol{\beta}_{T-1} \mathbf{X}_{ijT-1} + \sum_{t=0}^{T-2} \left(\boldsymbol{\beta}_t \mathbf{X}_{ijt} \right) + \boldsymbol{\varepsilon}_{ijT-1} \right] + \boldsymbol{\varepsilon}_{ijT}$.

Considering (A10), then, equation (A8) can be re-written as:

$$\text{(A11)}\quad \mathbf{A}_{ijT} = \boldsymbol{\beta}_T \mathbf{X}_{ijT} + \frac{\boldsymbol{\beta}_{IT}}{\boldsymbol{\beta}_{IT-1}} \mathbf{A}_{ijT-1} + \sum_{t=0}^{T-1} \left(\boldsymbol{\beta}_t \mathbf{X}_{ijt} + \boldsymbol{\varepsilon}_{ijt} \right) + \boldsymbol{\upsilon}_{i\,jT}$$

Hence, there is an endogeneity problem since $\mathbf{A}_{i\,jT-1}$ is correlated with $\boldsymbol{\upsilon}_{i\,jT}$, which contains $\boldsymbol{\varepsilon}_{ijT-1}$, a component of $\mathbf{A}_{i\,jT-1}$. Estimating equation (A11) using an OLS procedure may result in biased estimates.