

On Muslim Consumer Behaviour: A Mathematical Set-Up

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This paper attempts to set up a general mathematical framework in which the Muslim economic agent is entitled to move. The model presented in the paper highlights the construction of the general structure and its main components, as well as the relationships between those components. In particular, it examines the link that exists between the utility function and the *Fallah* function via the *Iman* level. We have also explored how the level of *Iman* could affect the relationship between the utility function and the *Fallah* function. Finally, the paper points out some of the remaining issues that must be addressed.

(i) INTRODUCTION

There has been a consensus among Islamic economists on the existence of some shortcomings in the Western economic theory of consumer behaviour and its unsuitability for explaining the behaviour of the Muslim consumers. Therefore, the behaviour of the Muslim consumers has been one of the subjects of research in Islamic economics in recent years. Researchers have attempted to make a theoretical framework for explaining the behaviour of the Muslim consumers which constitutes the basic foundation in modelling the structure of the economic system. One of the attempts made in this regard is the study by Zarqa (1980). In this study he presents an analysis of the Muslim consumer behaviour towards the choice between worldly consumption and reward in the hereafter, assuming

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that the later is the final objective of the Muslim. According to Zarqa, the Muslim individual endeavours to maximise his reward in the hereafter through spending on worldly consumption within the limits imposed by his income constraint and through moderation between subsistence and extravagance.

Zarqa infers that the relation between worldly consumption and reward in the hereafter is a positive linear relation for income levels which are below the subsistence level while it becomes a negative relation for consumption levels which exceed extravagance. As within the range of moderation (for example between subsistence and extravagance levels); the relation between consumption and the reward in the hereafter is a negative one which can be represented diagrammatically with level curves that are convex to the origin.

Zarqa's contribution is a diagram of the relation between worldly consumption and the reward in the hereafter. The concept which he presents of this relation for consumption levels below subsistence and above extravagance is considered acceptable as it emanates from the explicit *Quranic* teachings which urge a Muslim to attain subsistence level and prohibit him from *israf* in consumption. The study alleges that an inverse non-linear relation exists between consumption and reward under the condition that consumption stays between levels of subsistence and extravagance. However Zarqa does not advance adequate explanation for its existence. Zarqa's study does not present a theory for the behaviour of the "rational" Muslim consumer which explains consumer preferences among consumption opportunities available for him and left the question open for future research. In the same direction of thought Naqvi (1981) introduced some of the distinguishing characteristics of the Muslim consumer which are summarized as follows:

- (1) The Muslim consumer's consumption set is different from the usual consumption set since it is restricted to *Tayibat* only.
- (2) A Muslim consumer will not be totally "insatiable" with respect to every commodity.
- (3) The utility of the Muslim consumer depends on the utilities of other individuals in the society so that his behaviour is not selfish.

From the standpoint of Naqvi, the consumer maximises his utility by choosing between different combinations of goods permissible by Allah (S.W.T) with his income limits on condition that he does not exceed the prodigality (*Israf*) limit taking into consideration the utilities of other individuals in the society. The Muslim consumer thus faces two types of constraints:

- (a) The income constraint, and
- (b) The religious constraint

This approach was criticised as an additional constraint on the behaviour of the Muslim consumer (Zarqa, 1980). These teachings in fact came as directions to maintain the human nature of individuals and to realise goodness for them in this world and reward in the hereafter, thus giving them more utility. Naqvi's study does not differ much from that of Zarqa and it does not discuss explicitly the axioms which shape the Muslim consumer behaviour towards choice between available alternatives. Apart from these two studies, Kahf (1980) presents the Muslim utility as a function of (a) quantities of goods and services consumed, and (b) spending for the sake of Allah (S.W.T). The consumer endeavours to maximise the utility he gets from the consumption of goods and services according to what religious teachings permit and also from spending for the sake of Allah (S.W.T) which can be defined generally as the spending from private wealth on others. Kahf's study discusses particularly the relation between *Iman* and that part of income the individual decides to spend for the sake of Allah (S.W.T.) and explains that there is a positive correlation between them. Another study which is worth-mentioning is that of Khan (1984). In this study the utility function of a Muslim individual depends on two arguments (a) spending to achieve satisfaction in this world which includes present and future consumption, and (b) spending for others with a view to earn reward in the hereafter.

This study makes clear that the division of income between (a) and (b) is left to the Muslim consumer and the proportions depend on the level of Taqwa of that individual. The author argues that the axiom of rationality required for the Muslim consumer behaviour is not simply an assumption that might or might not be true, but it is something that a Muslim has to learn and acquire from the teachings of Islam. The main elements of the behaviour of a Muslim consumer summarised by Khan's study are as follows:

- (1) A Muslim consumer's total spending is classified into two major categories; to achieve satisfaction in this world which includes present and future consumption and spending for others with a view to earn reward in the hereafter.
- (2) The consumption set of a Muslim includes only *halal* good.
- (3) The allocation between worldly spending and spending for others is left to the rational consumer behaviour which depends on *Taqwa*.
- (4) The degree to which a Muslim consumer fears God is an essential parameter in the determination of his behaviour.
- (5) There is only an inferior limit to the spending for others for those who are Islamically eligible.
- (6) A Muslim is allowed to save but a major part of his savings will have to be invested; otherwise *Zakat* will erode these over time.

Khan has succeeded in setting up the general Islamic framework that governs the Muslim consumer behaviour. In his study on implications of Islamic consumption patterns for savings, growth and distribution, Khan uses the Islamic norms to explain the Muslim consumer behaviour in allocating his spending between spending for the sake of Allah (S.W.T) and worldly spending. He succeeds in putting forward the differences between the Islamic norms and those which govern the consumer behaviour in the secular economy. Even though these studies have tried to lay down the main Islamic premises that define the framework in which the Muslim consumer behaves, none of these studies has explicitly tackled the axiomatic problem of preference-ordering in the different consumption subsets. It is well-known that the most common way to derive the demand function is through maximising the utility function subject to the budget constraint. The properties of the demand function represent the nature of the utility function and the form of the utility map. The utility map is determined by the preference-ordering of the consumer and the different axioms that govern these preference ordering. It is well-known that these axioms in fact represent the cultural, moral and

philosophical foundations of the Western society. The question is: can these axioms, which represent the backbone of the secular economy, be used to explain the behaviour of the Muslim economic agent? The answer is of course not. It is then crucial for Muslim economists to try to formulate the axioms that explain the preferences of the Muslim consumer. A study to look at the mathematical formulation of the axiomatic problem of the preference-ordering of the Muslim consumer has been prepared by B. Bendjilali & Bachir (1989). First, they divide the Muslim consumption set into three distinct subsets, the *Tharouriyat* subset, the *Ihtiyajiyat* subset and the *Kamaliyat* subset. This division is based on the opinions of Muslim scholars. Second, the study presents two main axioms which clarify the preference-ordering relationship of the Muslim consumer with goods. According to the first axiom, the preferences of the Muslim consumer have to go through the following three phases. First, the consumer gives a preference to the subset of *Tharouriyat*, then to the subset of *Ihtiyajiyat*, and lastly, to the subset of *Kamaliyat*. However, the second axiom, which is considered as a weak axiom, allows the Muslim consumer to choose between goods in two steps. First, he begins by choosing between the *Tharouriyat* goods, then he will choose between the *Ihtiyajiyat* goods, and *Kamaliyat* goods considering these two latter categories of goods as forming one common subset. Based on these two axioms, the study derives the Muslim utility map and finds a clear difference between this latter one, and the usual utility map. In addition, the study points out the differences in the properties of the utility function of the Muslim consumer and the utility function of the non-Muslim consumer. Finally, the study has succeeded in introducing the *Thawab* variable as a built-in argument in the utility of the Muslim consumer, which makes it different from the non-Muslim one. In conclusion, the paper argues that the demand of the Muslim consumer will be different from the secular one since the utility map is different.

Even though these studies have made some real progress in the axiomatic setting and mathematical formulation of the Muslim consumer, the theory of the economic agent in general and of the Muslim consumer in particular is still at its first stage. Contrary to the consumer in the secular economy, who is motivated by self interest and non-satiation principles, the Muslim consumer is ordered to behave according to Allah's laws. In addition, his objective in this world is to worship Allah (S.W.T) as clearly stated in *Quran* [Az-

Zariyat : 56]. He is taught to behave in concordance with the Islamic Shariah in all his actions taking into account the limits prescribed in *Quran* such as the limits of prodigality (*Israf*) in consumption. He is also taught to consume *Tayibat* goods, to care about the needy and the neighbours, and to spend part of his wealth in the way of Allah (S.W.T) [Al-Isra'a : 26]. Islam goes beyond this by teaching Muslims to spend for others irrespective of worldly benefits, while in the case of the secular economy, the primary motives for the economic agent are worldly and material rewards which take various forms such as security, prestige, better social position, etc. This teaching is clearly stated in *Quran* [Al-Insan : 8-10) "*And they feed, for the love of Allah, the indigent, the orphan and the captive. (Saying) we feed you for the sake of Allah alone: No reward do we desire from you, nor thanks. We only fear a Day of frowning and distress from the side of our Lord*". Hence, the utility function of the Muslim consumer depends on his own consumption as well as on the consumption of others. The direction of increase in the desires (satisfactions) of a given economic agent might be guided by either the vector of profits, or by the vector of revenue, etc. depending on what gives him the most satisfaction. However, a Muslim economic agent, and in particular, the Muslim consumer is guided by the *Fallah*. In other words, he is guided by the vector of actions that give him more reward. He should then choose his actions that are closer to the *Shariah* since they are optimal in the Islamic sense. However, the desires of any economic agent are in some circumstances in opposite direction to the *Fallah* as it is clearly mentioned in the *Quran* [Al-Baqarah: 216] and [Al-Maidah: 48-49] among others.

"Fighting is prescribed upon you and ye dislike it. But it is possible that ye dislike a thing which is good for you and that ye love a thing which is bad for you"

[Al-Baqarah: 216].

"To thee we sent the scripture in truth confirming the scripture that came before it and guarding it in safety, so judge between them by what Allah hath revealed and follow not their vain desires diverging from the truth that hath come to thee. And this (he commands), Judge thou between them by what Allah hath revealed and follow not their vain desires" [Al-Maidah: 48-49].

As it has been stated in the above verses, desires (satisfaction) of an economic agent might diverge from the truth and hence lead to a negative *Fallah* (reward) even though it adds temporary (instantaneous) satisfaction to him. Therefore, the utility of a Muslim consumer might increase, in terms of worldly satisfaction in some cases, while his *thawab* decreases. Hence, the direction of increase in satisfaction in some situations might be in opposite direction to the increase in reward. These cases might happen because of the low level of *Iman*, the degree of attraction, the ignorance of *Din*, etc. Accordingly, some questions arise which are of interest and must be looked at; for instance, how to interrelate the utility concept to the *Fallah* concept? What is the general mathematical framework that is suitable for the Muslim economic agent in general and the Muslim consumer in particular? The main objective of this paper is to try to answer these questions. The plan of this paper is as follows. Section (ii) discusses the mathematical formulation of the Muslim economic agent, introducing the Muslim consumer as a particular case. Section (iii) examines the relationship between the utility function and the *Fallah* function. Section (iv) summarises the study and raises some questions for further research.

(ii) GENERAL MATHEMATICAL FORMULATION OF THE MUSLIM CONSUMER:

This section is concerned with the mathematical set up of the Muslim economic agent in general and treats the Muslim consumer in particular. We shall first define the main components of such a framework as well as their interrelationships. We shall follow the notations and framework developed by Hurwicz (1960) and Sato (1981) and shall extend the framework to include the Muslim objective. We shall pinpoint the most important issues related to the Muslim consumer that must be resolved.

Let us consider N = economic agents which can be either consumers or producers. Let e_i denote the characteristics of the i^{th} agent.

$$(1) \quad e_i = (C_i; \succeq_i; w_i; \text{production, Technology, } dL_i)$$

where C_i denote the consumption possibility set of agent i ; \succeq_i his preference ordering, w_i the vector of endowments of private

goods and dL_i the degree of his *Iman*.

Let E_i denote the class of all possible characteristics of the i^{th} agent or the space of a priori admissible characteristics of the i^{th} agent ($i=1,2,\dots,N$). Let us assume that the environments are non-decomposable and let E denote the class of all possible environments. E can be written under this assumption as a Cartesian Product of each agent's class of characteristics. That is:

$$(2) \quad E = \prod_{i=1}^N E_i$$

The economic agents communicate the information available between themselves according to Islamic injunctions which are defined by the *Shariah*. The process of communication can be thought of as an interactive process and described by response functions. The response of each Muslim agent determines messages to be sent by that agent as a function of the previously received message as well as of the economic environment. These messages are of course subject to the moral values of that same agent, that include his accumulated knowledge of Islamic *Shariah* as well as his ability to practically endorse these Islamic values in his daily life. After some stages of iterations are carried out according to the Islamic *Shariah*, agreements may be reached. In other words, equilibrium or stationary messages are obtained. Then a joint action is taken on the basis of the equilibrium messages. Hence, the process can be decomposed into two stages. In the first stage, a 'message correspondence' is defined which associates a message to a vector of characteristics. That is, the correspondence takes the space of environments E as its domain and the 'message space' denoted by M as its range. In the second stage, a new function is defined which associates an equilibrium message to a given joint action. In other words, this new function called 'outcome function' is defined from the space of messages M into the 'space of actions' denoted by A . This space of joint actions which include exchanges, production, etc. plays an important role in the sense that all "good" and "bad" actions are included in it and the final outcome is based on these actions. On the other hand, these two stages can be considered as one stage, whereby, initially the Muslim economic agents are endowed by their own characteristics and come to exchange

their endowments to arrive at a new resource allocation through an 'Islamic Process' that gives them the highest level of satisfactions. Hence, a resource allocation process can be defined as a model describing how decisions for joint actions are taken in a given environment.

Definition [Hurwicz]:

A resource allocation process is a correspondence from the set of environment to the set of joint actions which will be called performance correspondence. Formally we have the following diagram:

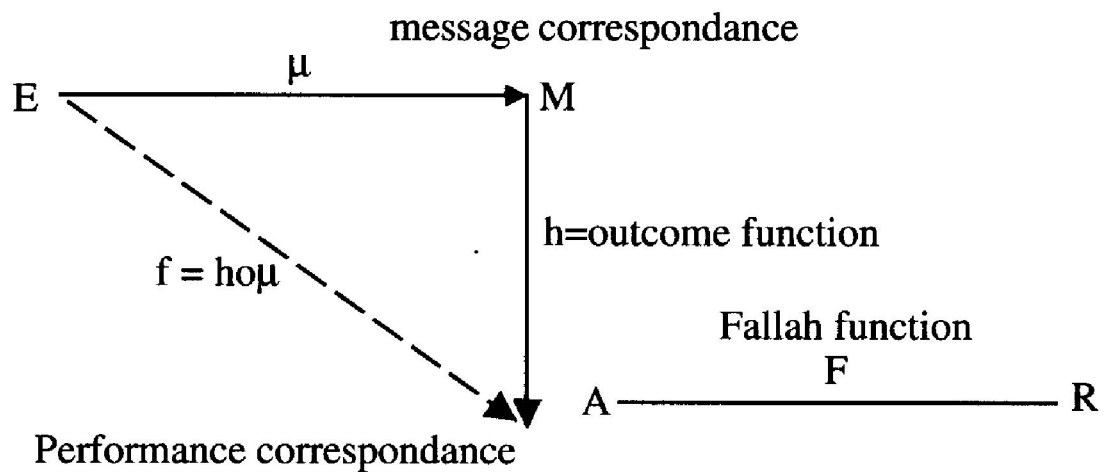


Figure 1

where E , M , A and R denote respectively the space of characteristics, the space of equilibrium messages, the space of joint actions and the space of (real numbers) the correspondences; μ , h , f and F denote respectively the message correspondence, the outcome function, the performance correspondence and the *Fallah* function. Some of the well-known processes are the perfectly competitive process¹, the Hurwicz's greed process, and the Lindahl process². These processes possess certain properties such as non-wastefulness, and privacy preserving over a class of environments³.

¹ See Koopmans [5] for the non-wastefulness of the perfectly competitive process (P.C.P).

² See Milleron [6].

³ For P.C.P. processes over environments free from indivisibility, externalities, public goods, non convexities: For Hurwicz Greed process over a wide class of decomposable environments the Lindahl process is acting over a class of environment with public goods.

It is easy to notice from Fig. 1 that the message correspondence acts on the space of characteristics into the space of stationary messages. The image of a vector of characteristics by the message correspondence is in general a proper subset of the message space M .

Let us assume, for simplicity, an economy with two consumers, two private goods, one public good and only one producer.

Let

$X_i = (X_i^1, X_i^2)$ = Net exchange of private goods by the i^{th} consumer ($C_i = 1, 2$)

$Y = (Y^1, Y^2)$ = Net exchange of the private goods by the producer

Z_i = consumption of the public good by the i^{th} consumer ($C_i = 1, 2$)

V = production of the public good.

We assume that the consumers are net demanders, that is, X_i is positive and that the producer is a net supplier (i.e. $Y > 0$). Then, $a_i = (X_i, Z_i)$ defines the actions of the i^{th} consumer and $a_p = (Y, V)$ defines the actions of the producer. Hence, $a = [(a_1, a_2); a_p]$ defines a joint action. In order for this joint action to be consistent we must have the following conditions satisfied.

$$(3) \quad X_1 + X_2 = Y$$

$$(4) \quad Z_i = V \quad i = 1, 2$$

This joint action has been the result of a series of stages which could be accomplished either by the competitive market process or by the Hurwicz's Greed process. These stages include, in general, the formation of prices, the behaviour of the market, the behaviour of the consumer, etc. However, the final objective of the Muslim economic agent, in general, and of the Muslim consumer, in particular, is to maximise his *Fallah* or reward through an allocation process governed by Islamic norms. The *Fallah* function acts on the space of joint actions having the set of real numbers as its range. In other words, the *Fallah* function associates to a given joint action a real number. The outcome, in our context, depends on how the Islamic process has been along the road. Therefore, if ' a ' denotes the vector of joint actions realized under the strict application of the *Shariah* norms by the Islamic process then the value of the *Fallah* acting on a joint action ' a ' will be influenced by the 'distance' or closeness of the joint action ' a ' to the joint action ' a^s '. In other words, how much *Fallah* does a

Muslim consumer gets from his action will depend on how close his action has been to the 'best' action. Hence, we need to define a distance or metric on the space of joint actions. In Mathematical language, we need to define topology on the space of joint actions A which permits us to define a norm and hence a distance. Let us assume that this problem has been settled and let " d " denotes the metric on the space of joint actions A . The 'best action' (according to Islamic *Shariah*) ' a^s ' can be written as: $a^s = (a_1^s, a_2^s)$ where a_1^s and a_2^s represent respectively the 'best' action for the representative if Muslim consumer and the representative Muslim producer.

Let

$$(5) \quad d_i = d(a_i; a^s) \text{ and } d_j = d(a_j; a^s)$$

Represent the deviations between a_i and a^s and a_j and a^s where a_i and a_j represent respectively the actions of the i^{th} and j^{th} consumer. Let A_i be the space spanned by the vector ' a_i '. Let Π_i denote the projection from the space of joint actions A into A_i , then we define the *Fallah* function associated with the i^{th} economic agent by the following diagram

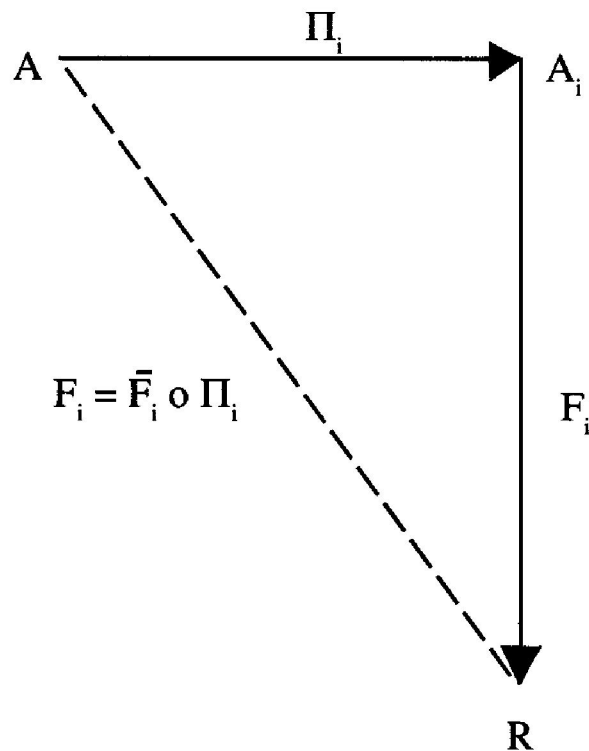


Figure 2

$$(6) \quad F_1(a) = \bar{F}_1(a_1, a_2, a_p) = (\bar{F}_1 \circ \Pi_1)(a_1, a_2, a_p) \\ = F_1[\Pi_1(a_1, a_2, a_p)] = F_1(a_1)$$

and similarly for $i = 2$ and $i = p$

Using equation (5) and Fig. 2, we deduce the following relationship.

$$(7) \quad d_i < d_j \text{ if and only if } F_i(a) > F_j(a)$$

The relationship (7) shows that if d_i is less than d_j , that is, if i^{th} consumer's action is closer to the best action ' a_1 ' defined according to the *Shariah* than the j^{th} consumer's action, then the *Fallah* of the i^{th} consumer is greater than the *Fallah* of the j^{th} consumer. In the next section, the study will look at the existence of the relationship between the utility function and the *Fallah* function.

(iii) ON THE RELATIONSHIP BETWEEN THE UTILITY FUNCTION AND THE *FALLAH* FUNCTION:

The objective of this section is to try to materialize the link that exists between the *Fallah* and the utility function. For this let U_i denote the utility of the i^{th} economic agent. U_i acts on the space of actions and associates with each joint action a positive real number. Let us now complete the diagrams given in Fig. 1 and 2 as follows:

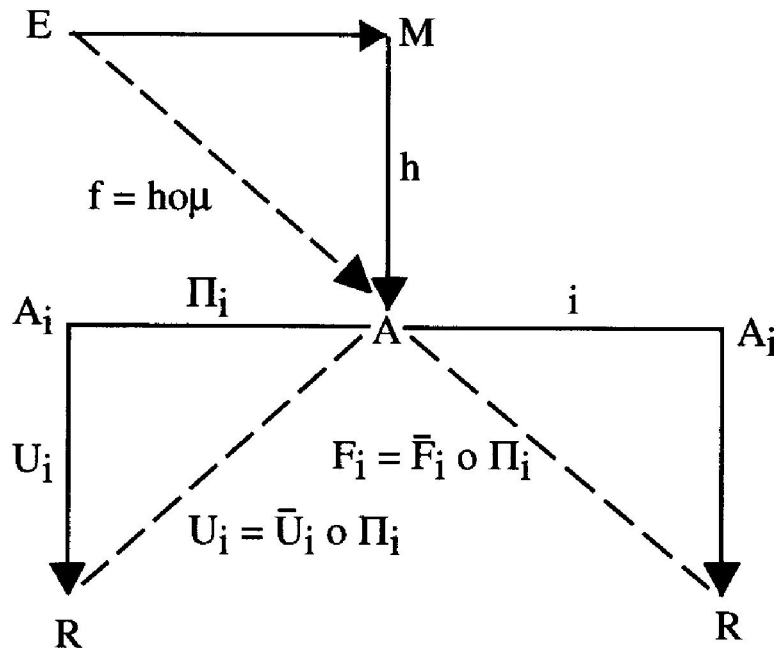


Figure 3

For instance, for $i = 1$ (consumer 1) we have

$$(8) \quad \begin{aligned} u_1(a) &= u_1[(a_1, a_2); a_p] = (u_1 \circ \Pi_1)[a_1, a_2, a_p] \\ &= u_1[\Pi_1(a_1, a_2, a_p)] = u_1(a_1) \end{aligned}$$

where u_i is the utility of the i^{th} economic agent restricted to act only on his own actions. After this brief mathematical setup, the problem left is how to relate the *Fallah* correspondence F_i to the corresponding utility function u_i .

As mentioned earlier in the introduction of this study, the increase in satisfaction (desires) of a given Muslim consumer might not be positively related with his *Fallah*. It might even occur that they are in opposite direction. It has also been argued in this study that the more *Mutaqi* a Muslim consumer is, the more satisfaction he gets from following the Islamic principles in his day to day life. On the other hand, the lower the *Iman* of the Muslim consumer, the less he is disposed to comply “internally” (self compliance) to the *Shariah*. For instance, he might get satisfaction by gathering wealth through *riba* and try to find a justification for it. A Muslim consumer whose *Iman* is so low might very well have a selfish conduct in his lifetime and as a result might get satisfaction out of his conducts (actions). However, these same conducts might result in negative *Fallah*. Hence, satisfaction (utility) and *Fallah* are correlated through the *Iman* level. A Muslim economic agent with high level of *Iman* chooses actions that permit him to move fastest (with the maximum speed) in the direction of increasing *Fallah*, and as a result get the maximum satisfaction. This fastest direction is expressed mathematically by the use of the gradient vector. Thus, we can express formally the following relationship between the *Fallah* and the utility of the i^{th} Muslim economic agent.

$$(9) \quad \bar{\nabla} U_i = \lambda(I) \bar{\nabla} F_i$$

Where $\lambda(I)$ is a parameter, depending on the level of *Iman*, I . The value of λ varies between minus one and plus one depending on the level of *Iman*. If the level of *Iman* (I) is at its maximum say (I_M) then the value of $\lambda(I_M)$ is equal to one. When the value of *Iman* is at its minimum ($I=0$) then $\lambda(0)$ is equal to minus one. Hence, from equation (9) we can deduce that when the level of *Iman* is at its

optimum then the utility of the Muslim agent is in perfect positive correlation with his *Fallah*. In other words, the behaviour of the Muslim is in this case in perfect harmony with the Islamic *Shariah*. His desires (utility) are governed by Islamic norms which he accepts to be an integral part of his behaviour. On the other hand if his level of *Iman* is at its minimum then his behaviour does not go along with most of the Islamic *Shariah*. He always tries to satisfy his desires (getting the maximum worldly satisfaction possible out of his actions) regardless of the outcome of his *Fallah*. This is so because most of the Islamic injunctions are not a built-in part of his personality (behaviour), and consequently he is Islamically short-sighted. The two particular cases mentioned above can be formulated as follows: First if the level of *Iman*, I , is equal to its maximum then equation (9) becomes

$$(10) \quad \bar{V} U_i = \bar{V} F_i$$

Equation (10) shows that the utility gradient vector is parallel to the *Fallah* gradient vector of the i^{th} Muslim economic agent and going in the same direction. On the other hand, if the *Iman* level is at its minimum then equation (9) becomes

$$(11) \quad \bar{V} U_i = -\bar{V} F_i$$

Equation (11) expresses the fact that the utility gradient vector is exactly in opposite direction to the *Fallah* gradient vector. In this case the maximization of the utility function leads to a minimization of the *Fallah* function. However, in the former case, the maximization of the utility function corresponds to the maximization of the *Fallah* function.

(iv) CONCLUSION:

This article has succeeded in setting up a general mathematical framework in which the Muslim economic agent (consumer or producer) is entitled to move freely. The model presented in the study has emphasised the construction of the general structure and its main components. The paper has also examined the link that exists between the utility function and the *Fallah* function and finds that the *Fallah*

is related to the utility through the *Iman* level. We have arrived at the conclusion that the higher the level of *Iman* is, the closer the utility function is to the *Fallah* function.

This study has pointed out some of the main issues that must be resolved before pursuing this line of action. First, at the level of the space of a priori admissible characteristics (or class of all possible characteristics), the preference-ordering is included as one of the characteristics of the economic agent. This preference-ordering is in general based on axioms which themselves are derived from the philosophical foundations or the ideological principles of the society in which that economic agent lives. These axioms, which play an important role in the construction of the utility map, have to be sorted out and mathematically well formulated. Second, at the level of the space of actions, the mathematical problem of constructing topology has been raised. A solution to this problem permits to define a norm and hence a distance. It finally allows an ordinal comparison between *Fallah* of any two Muslim economic agents. Third, any joint action is the outcome of some process. The process which possesses some properties such as non-wastefulness, privacy-preserving, cooperativeness etc., has to be defined according to Islamic norms. Furthermore, consumers behavioural properties have to be looked at from an Islamic angle and then have to be formulated mathematically in a context that suits the Islamic *Shariah*.

NOTES

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