LINEAR VS. NONLINEAR POROSITY ESTIMATION OF NMR OIL RESERVOIR DATA

M. A. ABOU MANDOUR*, M. M. EL-AFFIFY*, M. H. HASSAN*°, A. M. ALRAMADY**

** Nuclear and Radiological Engineering Department, Alexandria University, 21415, Alexandria, Egypt*

*** Atomic Energy Authority, P.O.Box 13975, Abu Zabal, Egypt.*

mohsenmandour@yahoo.com, elafify@yahoo.com, mhmheg@yahoo.com, ameerkamel@yahoo.com

° Currently on Leave: Faculty of Engineering, King AbdulAziz University, P. O. Box 80200, Jeddah, 21589, Saudi Arabia

ABSTRACT Nuclear Magnetic Resonance (NMR) is widely used to assess oil reservoir properties especially those that can not be evaluated using conventional techniques. In this regard, porosity determination and the related estimation of the oil present play a very important role in assessing the economic value of the oil wells. Nuclear magnetic resonance data is usually fit to the sum of decaying exponentials. The resulting distribution; i.e. T_2 distribution; is directly related to porosity determination. In this work, three reservoir core samples (Tight Sandstone and two Carbonate samples) were analyzed. Linear Least Square method (LLS); using regularization parameter (α) to smoothen the solution; and non-linear iterative least square fitting; using Levenberg-Marquardt method; were applied to calculate the T_2 distribution and the resulting incremental porosity. The linear solution was used as the initial guess for the iterative nonlinear solution.

Linear modeling is usually used to extract T_2 information from NMR logging data in real time. In experimental NMR logging, the goal is how to get the best inversion of the data into T_2 distribution regardless of the time of analysis. In terms of porosity, linear modeling assumes that the pore sizes are pre-selected while nonlinear modeling, starting from a properly chosen initial value, predicts pore sizes in an iterative way to properly predict real pore size values. Thus, it is more favorable to use nonlinear models over linear models. However, the order of magnitude of time needed for the linear solution is in the range of few minutes while it is in the range of few hours for the nonlinear solution.

Parametric analysis for the linear vs. nonlinear methods was performed to evaluate the impact of number of exponentials, and effect of the regularization parameter on the smoothing of the linear solution. Effect of the type of solution on porosity determination was carried out. Twelve exponentials were used for both the linear and nonlinear solutions. It was shown that the linear solution begins to be smooth at $\alpha = 0.5$ which corresponds to the commonly used industry value. Regardless of the fact that small differences exist between the linear and nonlinear solutions for the three samples, these small values make an appreciable difference in porosity. The nonlinear solution predicts 12% less porosity for the tight sandstone sample and 4.5 % and 13 % more porosity in the two carbonate samples respectively.

KEY WORDS: *NMR, T_2, Nonlinear, and Porosity*

1. INTRODUCTION

During the past decade, the remarkable technology of nuclear magnetic resonance (NMR) logging has been improved continually and has been adapted for improved downhole assessment of formation pore and fluid properties [1].

In NMR, the entire pulse sequence - a 90° pulse followed by a long series of 180° pulses - is called a Carr-Purcell-Meiboom-Gill Pulse Sequence (CPMG sequence). T_1 is the time at which the magnetization reaches 63.32% of its final value. The time constant of the transverse magnetization decay is called the transverse relaxation time, referred to as T_2. The raw data obtained from the NMR logging are the spin-echo trains. Fig. 1 shows the time domain data of the NMR tool.

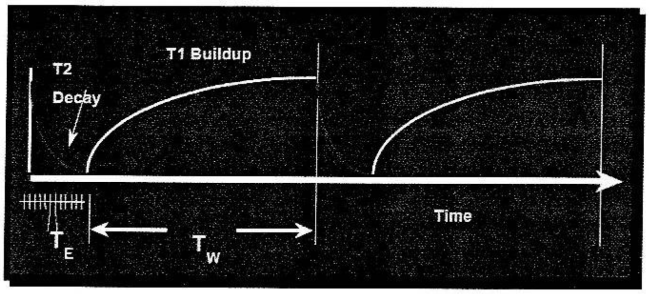


Fig. 1: Time domain data of the NMR tool

The amplitude of the spin-echo train at time *t*, which is the amplitude of the transverse magnetization *MX (t)*, is given by Ed. (1) [2]:

$$M_X(t) = M_{0X}e^{-t/T_2} \tag{1}$$

where M0X is the magnitude of the transverse magnetization at $t = 0$ (the time at which the 90° pulse ceases). The T_2 decay from the formation contains most of the petrophysical information obtainable from NMR logging and therefore is the prime objective of NMR logging measurements.

The time domain (actual) data measured in an NMR experiment are represented by a number of points separated by time *TE* (called the echo spacing) where the number of points equals the number of echoes. These data must be inverted into the T_2 domain in order to interpret the T_2 distribution of the sample [3]. The single decay of the echo data can be characterized as being made up of a continuous distribution of decays with T_2 as the decay constant. The data from an NMR experiment *y(t)* can be written as shown in Eq. (2):

$$y(t)=\left[\int_{T_2}^{T_2^{bulk}} P(T_2)e^{(-1/T_2)}d(T_2)+\varepsilon(t)\right] \tag{2}$$

where $P(T_2)$ is the T_2 distribution; and e^{-t/T_2} represents a continuous convolution of exponential decays from $T_2 = 0.0$ to $T_2 =$ the bulk relaxation of water; and $\varepsilon(t)$ is a vector that represent the noise in the data. The echo amplitudes are influenced by pore size distribution, diffusion within the pore space and bulk fluid properties.

2. COMEERCIAL VS. EXPERIMENTAL T_2 ANALYSIS:

One of the most important steps in NMR data processing is to determine the T_2 distribution that produces the observed magnetization. This step, called echo-fit or maping, is a mathematical inversion process [2].

Normally, the T_2 distribution of rocks is a continuous function. However, to simplify fitting the echo train, the mapping process uses a multi-exponential model that assumes that the T_2 distribution consists of m discrete relaxation times T_{2i} as shown in Eq. (3):

$$P(t)=\sum_{i=1}^{m} a_i \exp(-\frac{t}{T_{2i}}). \tag{3}$$

By dividing the total time into certain number of bins (usually around 10); the amplitude of each decay curve is determined. The number of coefficients equals the number of used bins. The essence of the linear modeling is that the values of T_{2i} are pre-selected (for example, 0.5, 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024 ms ...) and the mapping process focuses on determining the decay coefficients ai's.

In water-saturated rocks, it is proven mathematically [2] that the decay curve associated with a single pore will be a single exponential with a decay constant proportional to pore size; that is; small pores have small T_2 values and large pores have T_2 values. At any depth in the wellbore, the rock samples probed by the NMR tool will have a distribution of pore sizes. Hence, the multi-exponential decay represents the distribution of pore sizes at that depth, with each T_2 value corresponding to a different pore size.

Extracting most information from the tool data is a difficult problem, because of the logging tool operates with a low signal-to-noise ratio to minimize total logging time, and the transformation from the time domain into T_2 relaxation domain is an ill posed problem giving many possible solutions [4].

In NMR porosity determination, the linear least square method is usually used to get the decay coefficients ai's. Linear modeling usually applies fast and robust algorithm and can be used to extract T_2 information from NMR logging data in real time. This requires combining the reservoir and operational expertise of data and consulting services with the superior technology development to improve hydrocarbon recovery, minimize operational risks, reduce capital and operating costs and increase overall asset value.

The main difference between linear and nonlinear porosity determination is that in the linear solution, the T_{2i} values are pre-selected (for example, 0.5, 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024 ms ...). Thus, the linear inversion algorithm focuses on finding the amplitude of the decaying exponentials. In terms of porosity, this means that the pore sizes are pre-selected. The values of the incremental porosity corresponding to each pore size are determined after the inversion. While in case of nonlinear inversion, both the incremental porosity and the pore size are determined in an iterative way starting from a properly chosen initial guess.

Nonlinear models generally require the solution of nonlinear equations which can not be found in closed form nor determined after a limited number of algorithmic steps. If the solution of nonlinear models deviates from linear models only by small margin, then many important characteristics of the linear model apply to nonlinear model as well. Thus, to determine a relevant approximate solution, it is necessary to begin from an initial point which is located sufficiently close to the desired solution.

It is worth noting that using nonlinear methods in analyzing the NMR relaxation data may result in predicting higher or lower porosity values relative to the linear methods. This is translated into more or less actual oil prediction. Thus, we must distinguish between commercial and experimental nuclear magnetic resonance logging in terms of cost. Two parameters need to be considered: time and oil in place “OIP” (i.e., the amount of crude oil that is estimated to exist in a reservoir and which has not been produced.).

In commercial NMR, the more time spent in analyzing the data, the higher the cost will be. For example the average cost in Egypt of drilling machine is about 150,000 L.E. per day [5]. This means that when logging tool is working, the well site answer must be robust, compatible with further analysis in the data center, and must be computed in real time, i.e., we must use linear modeling which consume extremely less time than nonlinear modeling.

In experimental NMR logging, the time is not the critical factor since the goal is how to get the best inversion of the data into T_2 distribution regardless of the time of analysis. In this case, it is more favorable to use nonlinear models over linear models.

3. T_2 ANALYSIS USING THE LINEAR LEAST SQUARE METHOD (LLS)

In this paper, we analyzed the difference between linear vs. nonlinear least square methods to calculate the T_2 distribution and the associated porosity for three types of porous reservoir core samples [6]. Some of the details of the three samples are shown in Table 1.

Table 1: Samples Description [6]

	Sample A	**Sample B**	**Sample C**
Material	Tight Sandstone	Tight Carbonate (double porosity)	Carbonate (single porosity)
Echoes Numbers	6667	10833	4167

The first step to get the T_2 distribution is to plot the values of the magnetic field measured as a function of time. This is performed for each sample as follows: The number of echoes (M) is multiplied by *TE* (i.e., the echo spacing which equals 600 microseconds for the three samples) to get the total time of the train which is plotted as the X-axis. The Y-Axis is the magnitude of the magnetic field measured at each time value of the time axis. Results are shown in Fig. 2. As expected, carbonate samples are characterized by slow decay rate compared with sandstone samples [2].

The T_2 curve is obtained by using the multi-exponential decay model. By dividing the total time into certain number of bins (usually around 10); the amplitude of each decay curve is determined. The number of coefficients equals the number of used bins [7].

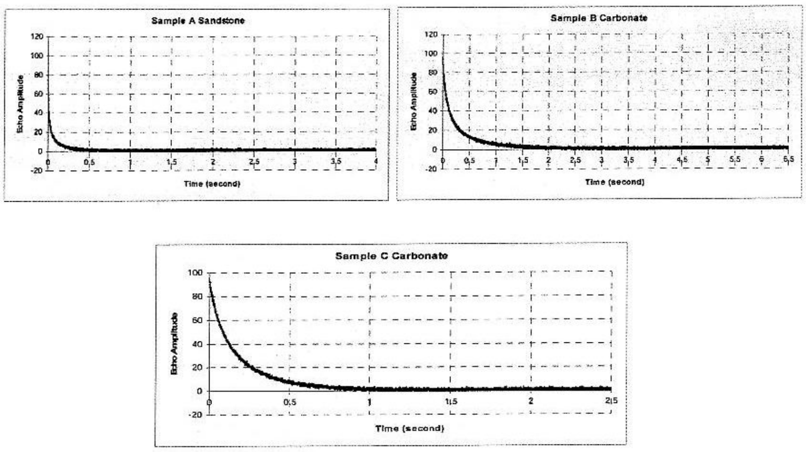


Fig. 2: The decay of the spin-echo train for the three samples A, B, and C respectively.

The least square method was used to get the decay coefficients ai's as mentioned in Eq.(3). MATLAB was used to solve the least square problems with a regularization parameter α. The Tikhonov solution is applied. Thus, instead of a straightforward inversion of the noisy data which produces widely oscillatory solution, the approach is to add a penalty function (or a regularization term α) to smooth or "regularize" the solution [3]. In case of NMR T_2 inversion, the best value of α depends on the apparatus used and on the signal to noise ratio. It is logical to choose an α where the standard error just starts to increase significantly. However, the choice is somewhat arbitrary. It can be anywhere from $\alpha = 0.1$ to slightly larger than 1 and it is usually around 0.5 [5].

3.1 Effect of changing the regularization parameter "α"

To estimate the impact of the value of the of regularization parameter "α" on the smoothing of the solution, we determined the linear solution for the NMR data of the three samples using different values of α; namely; 0.001, 0.5, and 10. As shown in Fig. 3, the solution begins to be smooth at $\alpha = 0.5$ which corresponds to the commonly used industry value [5]. In Fig. 4, the linear solution was obtained for a narrower range of the regularization parameter; namely; 0.05, 0.1, 0.3, 0.5 and 0.7. Moreover, the most favored value appears to be around 0.5 and above. In all subsequent calculations, we used a value of 0.5 for α.

3.2 Effect of Number of Exponentials on the Linear Solution

We studied the effect of assuming different values for the number of exponentials to fit the echo data of the three samples. The number of exponentials was taken to be 6, 12, 18, and 24. After some trial, these values were found to indicate clearly the difference between small and large number of exponentials. In Fig. 5 it is observed that 6 exponentials are not enough to obtain a good fit to the data and 12, 18, and 24 exponentials are almost the same. We can take the 12 exponentials as the optimum number of exponentials. As mentioned before, the usual industry trend is to choose 10 exponentials. Thus, using 12 exponentials is a good choice to be used in the analysis.

It should be mentioned that the number of exponentials nearly does not affect the time required to solve the problem in the linear solution. However, as will be clear from section 4, the number of exponential will affect the time needed to obtain the nonlinear solution. Since the initial guess for the nonlinear solution was taken from the linear solution, 12 exponentials seem to be acceptable to get a linear solution that can be compared with the nonlinear one.

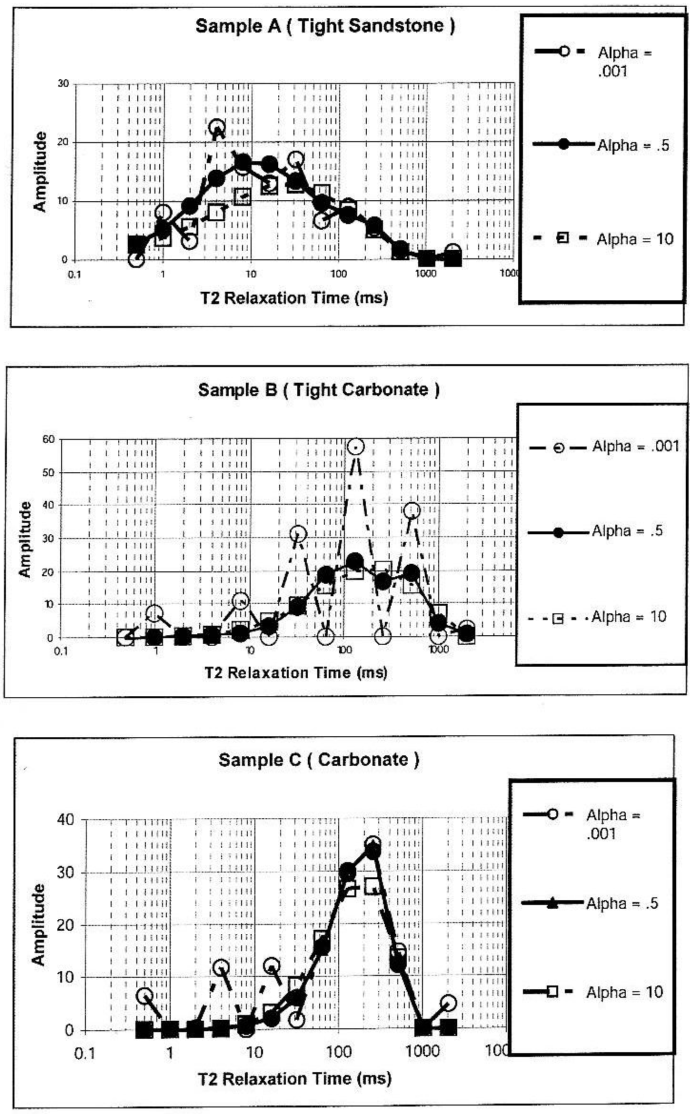


Fig. 3: Linear solution for different values of regularization parameter α: 0.001, 0.5 and 10 for the samples A, B, and C.

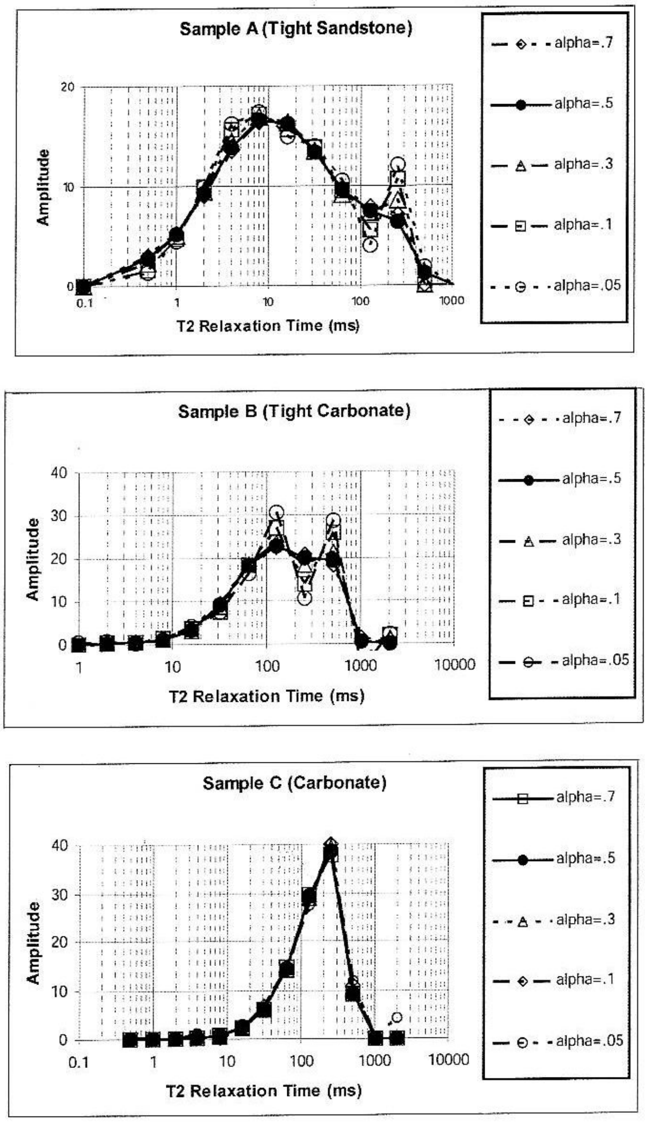


Fig. 4: Linear solution for narrower range of regularization parameter α: 0.05, 0.1, 0.3, 0.5 and 0.7 for the samples A, B, and C.

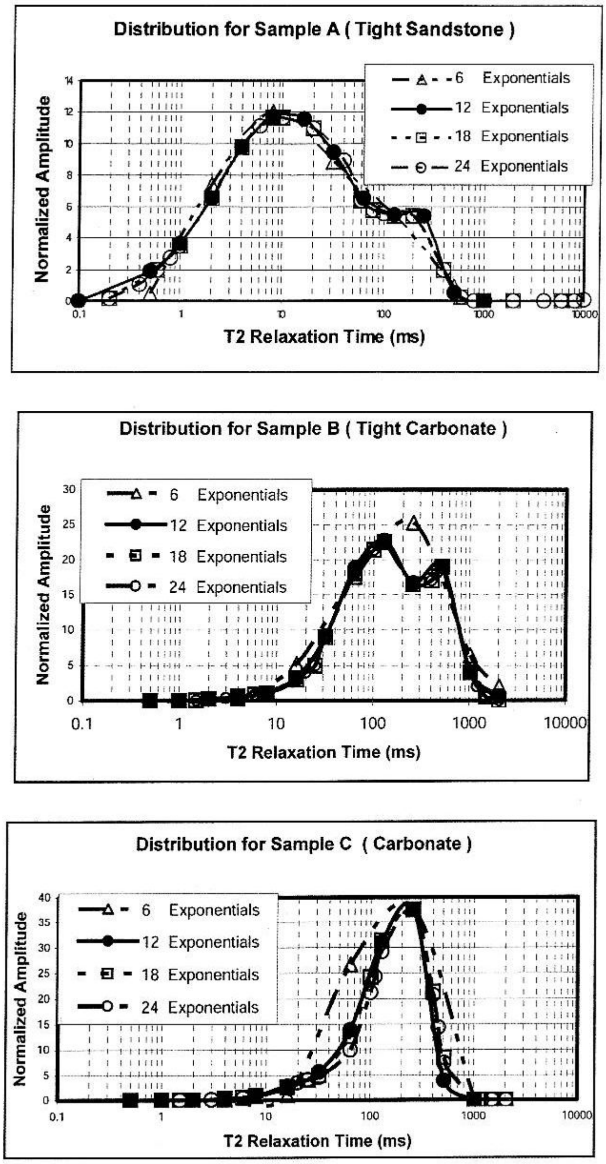
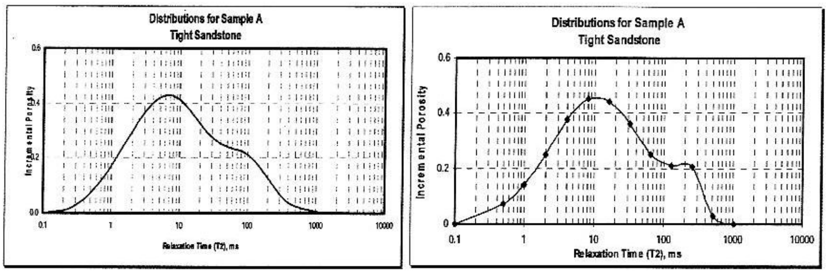


Fig. 5: T_2 distribution of sample A (Tight sandstone), sample B (Tight carbonate) and sample C (Carbonate) for different number of exponentials 6, 12, 18, and 24 using the linear model

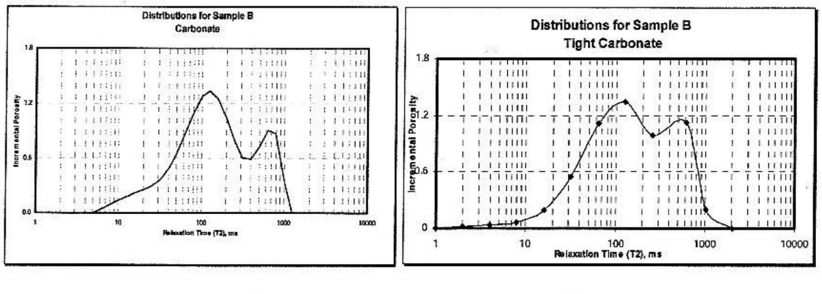
3.3 Incremental Porosity Using Linear Least Square Fitting Method

To check the validity of using the linear least square method, we determined the incremental porosity curves for the different samples. This was done by normalizing the amplitudes of the calculated T_2 distribution curves to the incremental porosity units by normalization of the maximum of the T2 distribution of each sample to the maximum of the original data. The calculated incremental porosity curves compare favorably with the original curves as shown in Figures 6, 7 and 8. As expected, the lowest porosity value was for the tight sandstone sample (A). The tight carbonate sample (B) showed less porosity relative to the carbonate sample (C).



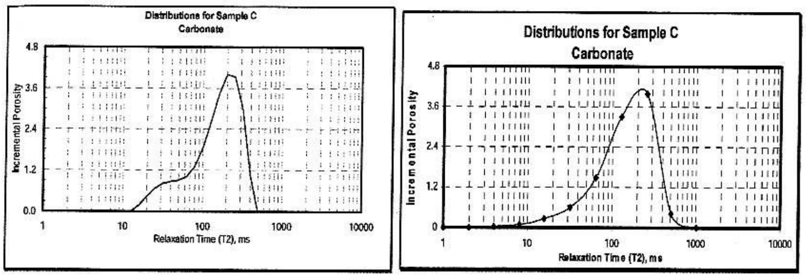
(a) (b)

Fig. 6: Porosity distribution curves (a) Original values and (b) Values obtained by using linear least square method for sample A (tight Sandstone)



(a) (b)

Fig. 7: Porosity distribution curves (a) Original values and (b) Values obtained by using linear least square method for sample B (tight Carbonate)



(a) (b)

Fig. 8: Porosity distribution curves (a) Original values and (b) Values obtained by using linear least square method for sample C (Carbonate)

4. NONLINEAR T2 ANALYSIS USING THE ITERATIVE LEVENBERG-MARQUARDT METHOD

We used the iterative least square method "Levenberg-Marquardt" [8-11] which is remarkably adaptive to the problem at hand. It is one of the most robust and certainly the most applied method for the solution of difficult problems in nonlinear modeling [8]. It is to be noted that nonlinear solution is ill posed and it is crucial to find a good starting values for the parameters [12]. In this work we used the results from linear solution as the initial guess of the nonlinear solution.

4.1 Effect of number of exponentials on the nonlinear solution

We studied the effect of assuming different values for the number of exponentials to fit the echo data of the three samples. The number of exponentials was taken to be: 6, 12, 18, and 24.

In Fig. 9 it is observed that 6 exponentials are not enough to obtain a good fit to the data and 12 exponentials are the optimum number. It is important to note that in case of nonlinear solution, the greater the number of exponentials the greater the time required to solve the problem. It is worth noting that the order of magnitude of time needed for the linear solution is in the range of few minutes while it is in the range of few hours for the nonlinear solution.

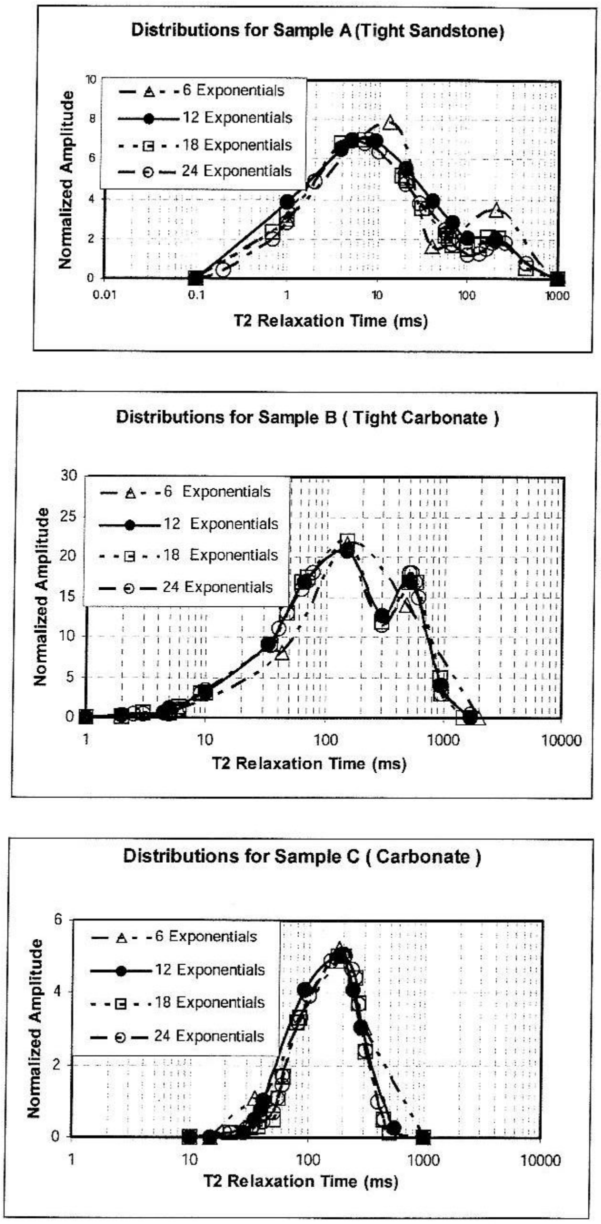


Fig. 9: T_2 distribution of sample A (Tight sandstone), sample B (Tight carbonate) and sample C (Carbonate) for different number of exponentials: 6, 12, 18, and 24 using the nonlinear solution.

5. NONLINEAR VS. LINEAR SOLUTION

In Fig. 10 we made a comparison between linear and nonlinear solutions considering the same number of exponentials “12”. It is to be noted that for the linear solution, the spacing of the T_2 relaxation times (corresponding to pore sizes) are equally spaced (on a logarithmic scale) while in the nonlinear model they follow the actual rock pores size distribution.

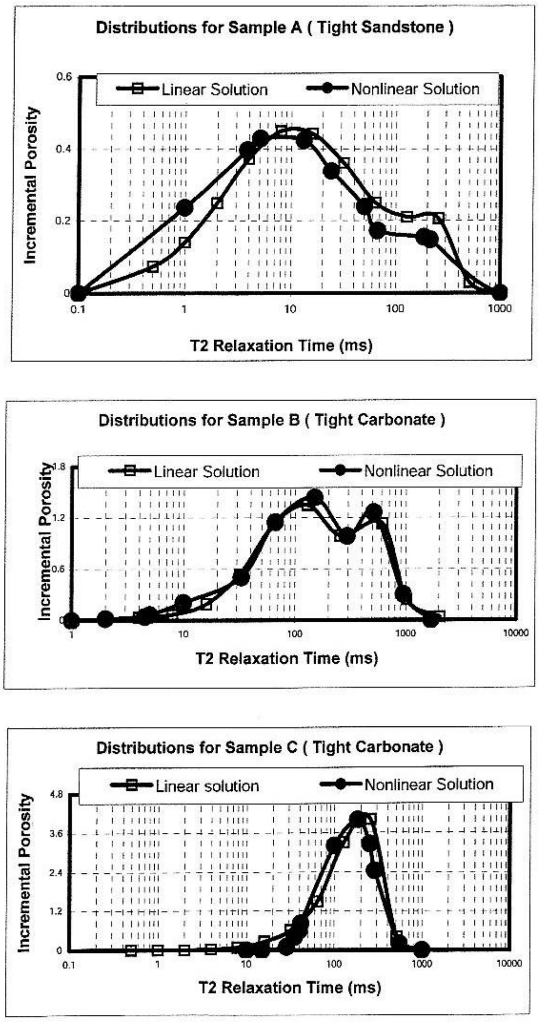


Fig. 10: A comparison between linear and nonlinear solutions using the same number of exponentials "12"

For the samples under investigation, small differences exist between the two solutions. This means that pre-selected pore sizes differ slightly from the actual pore sizes. However, these small variations make an appreciable difference in total porosity as shown in Table 2. The nonlinear solution predicts less porosity than predicted by linear solution by about 12% for sample A (Tight Sandstone). It also predicts more porosity in both samples B and C (Carbonate) by about 4.5% and 13% respectively.

Table 2: Percentage saving porosity calculated by using nonlinear solution over linear solution to predict the porosity for the three samples.

Sample	**Percentage saving of porosity by using nonlinear solution over linear solution**
Sample A (Tight Sandstone)	12 %
Sample B (Tight Carbonate)	-4.5 %
Sample C (Carbonate)	-13 %

To indicate the impact of the method of T_2 analysis on estimation of porosity, Table 2 shows the percentage of saving porosity calculated by using nonlinear solution vs. linear solution to predict the porosity of the three samples. If the porosity calculated from nonlinear solution is greater than the porosity calculated from linear solution, we refer to this as positive saving. On the other hand, if the porosity calculated from nonlinear solution is smaller, we refer to this case as negative saving. This difference in porosity estimation will be very useful to get more oil from the wells which mean more saving in the money paid. It is to be noted that these nonlinear calculations should be done for each sample to determine positive or negative porosity savings. Further analysis is needed to be able to predict whether the nonlinear vs. linear methods of T_2 analysis will yield positive or negative savings. This may be needed to be done for samples having different porosity distribution.

6 CONCLUSIONS

Three samples; namely, tight sandstone, tight carbonate and carbonate were used to assess the difference between linear and nonlinear T_2 inversion to get the incremental porosity. Comparison was made using 12 exponentials for the T2 inversion. The linear solution begins to be smooth at $\alpha = 0.5$ which corresponds to the standard industrial value for the regularization parameter.

The order of magnitude of time needed for the linear solution is in the range of few minutes while it is in the range of few hours for the nonlinear solution. For the T_2 distribution, small differences exist between the linear and nonlinear solutions. This means that pre-selected pore sizes differ slightly from the actual pore sizes for the three samples under consideration. However, these small variations make an appreciable difference in total porosity. Compared with the linear solution, the nonlinear solution predicts 12% less porosity for the tight sandstone sample and 4.5 % and 13 % more porosity in the two carbonate samples respectively.

Further analysis of the T_2 inversion using nonlinear methods is needed to examine if there is a range of pore sizes where the nonlinear methods are more efficient.

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