COUETTE FLOW PROBLEM FOR AN UNSTEADY MHD THIRD-GRADE FLUID WITH HALL CURRENTS

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ABSTRACT: In this work, we analyze Couette flow problem for an unsteady mangneto-hydrodynamic (MHD) third-grade fluid in the presence of a pressure gradient and Hall currents. Existing literature on the topic shows that the effecs of Hall current on Couette flow of an unsteady MHD third-grade fluid with a pressure gradient has not yet been investigated. The arising non-linear problem is solved by the homotopy analysis method (HAM) and the convergence of the obtained complex series solution is carefully analyzed. The effects of pressure number, Hartmann number and Hall parameter on unsteady velocity are discussed via analysis of plots.

ABSTRAK: Kajian dijalan untuk menganalisa masalah aliran Couette bagi bendalir MHD gred ketiga dan arus Hall. Bagi topik ini kesan arus Hall terhadap aliran Couette dalam bendalir MHD gred ketiga tak mantap dengan kecerunan tekanan, belum pernah dikaji selidik. Masalah tak linear berbangkit diselesaikan dengan kaedah analisis homotopi (HAM) dan ketumpuan solusi rangkaian kompleks dianalisa dengan teliti. Kesan nilai tekanan, nombor Hartmann dan parameter Hall terhadap halaju tak mantap diperbincangkan melalui plot yang dianalisis.

KEYWORDS: *Couette flow; hall currents; unsteady flow; third-grade fluid; HAM*

1. INTRODUCTION

In fluid mechanics, everyone is familiar with the Couette flow problem, the flow between two parallel plates in which bottom plate is fixed and upper plate is initially at rest and is suddenly set into motion in its own plane with a constant velocity, is termed as Couette flow [1,2]. Bhaskara and Bathaiah [3] have analyzed the Couette flow problem with Hall effects for flow through a porous straight channel. Ganapathy [4] has written a note on the oscillatory Couette flow in a rotating system. Erdogan [5] solved unsteady Couette flow for viscous fluid by the Laplace transform method. Stokes and Couette flows due to an oscillating wall have been discussed by Khaled and Vafai [6]. Hayat et al. [7] used the Laplace transform method to determine the analytic solutions of Couette flows of a second grade fluid. Oscillatory Couette flow has been studied by Singh [8]. Guria [9] discussed the Couette flow problem for rotating and oscillatory flow. Couette flow of an unsteady third-grade fluid with variable magnetic field has been investigated by Hayat and Kara [10], where the fluid is in an annular region between two coaxial cylinders. The axial Couette flow problem of an electrically conducting fluid in an annulus has been examined by Hayat et al. [11]. Das et al. [12] studied unsteady Couette flow problem in a rotating system. Recently, Zaman et al. [13] presented a solution for unsteady Couette flow

problem for the Eyring-Powell model. When a strong magnetic field is applied in an ionized gas of low density, the conductivity normal to the magnetic field is decreased by free spiraling of electrons and ions about the magnetic lines of force before suffering collisions. This phenomenon is known as the Hall effect and a current induced in a direction normal to the electric and magnetic fields is called the Hall current [14]. The study of the effects of Hall currents on flow of non-Newtonian fluids [15-23] is important because of its applications in power generators and pumps, Hall accelerators, refrigeration coils, electric transformers, in flight MHD, electronic system cooling, cool combustors, fiber and granular insulation, oil extraction, thermal energy storage, and flow through filtering devices.

In order to understand the interaction of electric, magnetic, and hydrodynamic forces in the unsteady third-grade fluid, we considered a simple flow problem, known as the Couette flow. The effects of a pressure gradient and Hall current on the flow are also taken in to account. The complex analytical solution for the non-linear problem is found by using the homotopy analysis method (HAM) [24- 31].

This solution is valid for all values of time in the whole spatial domain $0 \leq \eta < 1$. The convergence of the analytic solution is ensured with the help of the *ħ-curve*. The effect of pressure number, Hartmann number, Hall parameter, second-grade parameter and third-grade parameters on the unsteady velocity are illustrated through plots. Also, the effects of the pertinent parameters on the local skin friction co-efficient at the surface of the wall are presented numerically in tabular form.

2. FORMULATION OF THE PROBLEM AND ITS ANALYTIC SOLUTION

Consider the unsteady flow of an electrically conducting incompressible third-grade fluid between two parallel flat plates, subjected to a uniform transverse magnetic field. We assume that the bottom plate is fixed and the top plate is stationary when *t < 0* and at *t = 0*, the top plate starts moving impulsively in its own plane with a constant velocity *U* and a pressure gradient is also applied. The flow here is maintained by the motion of the top plate. The Cauchy stress tensor *T* for a third-grade fluid is given as [32]

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (tr A_1^2) A_1 \tag{1}$$

where *p* is the scalar pressure, *I* is the identity tensor, *μ* is the co-efficient of viscosity α_i, β_j, (*i = 1, 2* and *j = 1, 2, 3*) are the material parameters of third-grade fluid, and A_i (*i = 1, 2, 3*) are the first three Rivlin-Ericksen tensors defined by [32]

$$A_1 = (grad\ V)\ +\ (grad\ V)^T \tag{2}$$

$$A_{n+1} = \frac{dA_n}{dt} + A_n (grad\ V) + (grad\ V)^T A_n\ ,\ n = 1,2. \tag{3}$$

The equations governing the magneto-hydrodynamic flow with Hall effects are:

Velocity Field: $V = [u\ (y;\ t)\ ,\ 0,\ 0]$ (4)

Continuity equation: *div V = 0*   (5)

Equation of motion: $\rho \dfrac{dV}{dt} = div(T) + (J \times B)$ (6)

Maxwell equations: $div(B) = 0, \ curl\,(B) = \mu_m J, curl(E) = -\dfrac{\partial B}{\partial t}$ (7)

Generalized Ohm's law: $J + \dfrac{w_e\,\tau_e}{B_o} J \times B = \sigma(E + V \times B)$ (8)

Where $u(\,y;\,t)$ is the velocity component in the *x*-direction, *t* is time, $B\,(= B_0 + b)$ is the total magnetic field, B_0 is the applied magnetic field, *b* is the induced magnetic field, *J* is the current density, σ is the electrical conductivity of the fluid, *E* is the electric field, µm is the magnetic permeability, ρ is the fluid density, w_e and τ_e are the cyclotron frequency and collision time of the electrons respectively. We assume that, the quantities ρ, μ_m and σ are constants throughout the flow field, the magnetic field *B* is normal to the velocity vector *V* and the induced magnetic field is neglected compared with the imposed magnetic field, so that the magnetic Reynolds number is small [33]. We also suppose that $w_e\tau_e \sim$ O (1) and $w_i\tau_i << 1$ (where w_i and τ_i are the cyclotron frequency and collision time for ions respectively). Continuity Eqn. (5) is identically satisfied by the velocity taken in Eqn. (4). Under the aforementioned assumptions the unsteady Couette flow problem with Hall currents become;

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2(1+i\in)}{\rho(1+\in^2)}u + \frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial y^2 \partial t}$$

$$+ \frac{\beta_1}{\rho}\frac{\partial^4 u}{\partial y^2 \partial t^2} + \frac{6(\beta_2+\beta_3)}{\rho}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}$$ (9)

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y},$$ (10)

The boundary and initial conditions are

$$\begin{array}{llll} u(y,t) = 0, & at\ y = 0, & for\ t > 0, \\ u(y,t) = u_w = U, & at\ y = h, & for\ t \geq 0, \\ u(y,t) = 0, & at\ t = 0, & for\ 0 \leq y < h \end{array}$$ (11)

where *v* is the kinematic viscosity, $\in (= w_e\tau_e)$ is the Hall parameter, *U* is the velocity of the upper plate and *h* is the distance between two parallel plates and it will be considered as a length scale of the flow, eqn. (10) shows that *p* is independent of *y*. In order to non-dimensionalize the problem let us introduce the similarity transformations,

$$u = Uf(\eta,\xi), \eta = \frac{y}{h}, \xi = \frac{vt}{h^2},$$ (12)

where $f(\eta,\xi)$ is the dimensionless velocity function, η is the dimensionless distance from the bottom wall and ξ is the dimensionless time. Eqn. (9) and eqn. (11) become

$$f'' + \alpha\frac{\partial f''}{\partial \xi} + \beta\frac{\partial^2 f''}{\partial \xi^2} - \frac{N(1+i\in)}{(1+\in^2)}f + \varsigma f'^2 f'' - \frac{\partial f}{\partial \xi} - P = 0$$ (13)

$$f(0,\xi) = 0, f(1,\xi) = 1, for\ \xi > 0,$$ (14)

$f(\eta, 0) = 0, for\ 0 \le \eta < 1,$

where prime denotes differentiation with respect to η, $\alpha(= \alpha_1/\rho h^2)$ is dimensionless second-grade parameter, $\beta(= \beta_1 v/\rho h^4)$, $\varsigma\left(= \frac{6(\beta_2+\beta_3)U^2}{\rho v h^2}\right)$ are dimensionless third-grade fluid material parameters, $P\left(= \frac{h^2}{\rho U v}\frac{dp}{dx}\right)$ is the dimensionless pressure number and $N(= \sigma B_0^2 h^2/\rho v)$ is the dimensionless modified Hartmann number [33]. The local skin friction coefficient or fractional drag coefficient on the surface of the moving wall is

$$C_f = \frac{2\tau_{xy}|_{y=h}}{\rho u_w^2},\tag{15}$$

Now using equations (1), (2), (3), (4) and (12) the eqn. (15) can be written in dimensionless variables as

$$R_e \times C_f = 2\{f'(1,\xi) + \alpha\frac{\partial}{\partial\xi}f'(1,\xi) + \beta\frac{\partial^2 f'(1,\xi)}{\partial\xi^2} + \frac{1}{3}\varsigma(f'(1,\xi))^3\}\tag{16}$$

where $R_e(= Uh/v)$ is the Reynolds number. The boundary conditions (14) lead us to take base functions for velocity $f(\eta, \xi)$ as

$$\{\eta^n \xi^j/, n \ge 0, j \ge 0\}\tag{17}$$

The velocity $f(\eta, \xi)$ can be expressed in terms of base functions as

$$f(\eta, \xi) = \sum_{n=0}^{\infty}\sum_{j=0}^{\infty}\Omega_n^j \eta^n \xi^j\tag{18}$$

To start with the homotopy analysis methods, due to the boundary conditions (14) it is reasonable to choose the initial guess approximation

$$f_0(\eta, \xi) = \eta(1 + \xi - \eta\xi), \xi > 0\tag{19}$$

and the auxiliary linear operator

$$\mathcal{L}(f) = \frac{\partial^3 f(\eta, \xi)}{\partial\eta^3},\tag{20}$$

Following the HAM and trying higher iterations with the unique and proper assignment of the results converge to the exact solution:

$$f(\eta, \xi) \approx f_0(\eta, \xi) + f_1(\eta, \xi) + f_1(\eta, \xi) + f_2(\eta, \xi) + \cdots + f_m(\eta, \xi)\tag{21}$$

At $\in= 0.1$, *N=0.1, α=0.1, β=0.1, ς=0.1, P= 0.1* using the symbolic computation we obtained

$$f_1(\eta, \xi) = \eta(1 + \xi - \eta\xi) + \frac{3\hbar\eta\xi}{10} - \frac{3\hbar\eta^2\xi}{10}$$

$$- \frac{\hbar\eta\xi^2}{15} - \frac{\hbar\eta^2\xi^2}{10} + \frac{\hbar\eta^3\xi^2}{6} + \frac{13\hbar\eta\xi^3}{15} - \frac{11\hbar\eta^3}{10}$$

$$+ \frac{3\hbar\eta^3\xi^3}{10} - \frac{\hbar\eta^4\xi^3}{15} + \left(\frac{227}{1515} + \frac{i}{606}\right)\hbar\eta\xi^4 - \frac{\hbar\eta^2\xi^4}{20}$$

$$-\left(\frac{5}{303}+\frac{i}{606}\right)\hbar\eta^3\xi^4 - \frac{\hbar\eta^4\xi^4}{12} + \left(\frac{5}{606}+\frac{i}{1212}\right)\hbar\eta\xi^5$$

$$-\left(\frac{5}{303}+\frac{i}{606}\right)\hbar\eta^3\xi^5 + \left(\frac{5}{606}+\frac{i}{1212}\right)\hbar\eta^4\xi^5, \tag{22}$$

Similarly $f_2(\eta,\xi), f_3(\eta,\xi), f_4(\eta,\xi)$ and so on are calculated. The obtained values of $f_0, f_1, f_2,$ ----- lead us to take

$$f_m(\eta,\xi) = \sum_{n=0}^{2m+2}\sum_{j=0}^{3m+3-n} \Omega_{m,n}^j \eta^n\,\xi^j. \tag{23}$$

The total complex analytic solution in compact form is as

$$f(\eta,\xi) = \sum_{m=0}^{\infty} f_m(\eta,\xi) = \lim_{M\to\infty}\left(\sum_{m=0}^{M}\sum_{j=0}^{3m+3} \Omega_{m,0}^j \eta^n\,\xi^j\right) \tag{24}$$

where from initial guess in eqn. (19) we obtain

$$\Omega_{0,1}^0 = 1, \Omega_{0,1}^1 = 1, \Omega_{0,2}^1 = -1$$
$$\Omega_{0,n}^j = 0, \ (j = 0,1,2,3), \ (n = 0,1,2) \tag{25}$$

all other unknown constants can be determined by utilizing eqn. (25) and using the recurrence relations, which we have calculated but it is not possible to write here due to their length. We know that the auxiliary parameter $\hbar$ gives the convergence region and rate of approximation for the homotopy analysis method. From $\hbar$-curve in Fig.1 we note that the range for the admissible value for $\hbar$ is $-1.5 < \hbar < 0$. Our calculations depict that the series of the dimensionless velocity in eqn. (25) converges in the whole region of η and ξ for $\hbar = -0.5$.

3. GRAPHS, TABLES AND DISCUSSION

The discussion of emerging parameters on the dimensionless velocity $f(\eta, \xi)$ is as follows:

Figures 2 to 7 are plotted in the absence of Hall currents and in Fig. 8 the Hall current is taken into account.

Figure 2 presents the velocity profile f for various values of ξ. This figure shows that with the passage of time the velocity of the fluid increases due to continuous motion of the upper plate. The boundary layer thickness decreases with the passage of time and shear thinning is seen. Figure 3 elucidates the variation of Hartmann number on the velocity. It is found that the velocity increases with an increase in N and the boundary layer thickness decreases. This means that the magnetic force provides a mechanism to the control of boundary layer thickness. Figure 4 illustrates the influence of a second-grade parameter on the velocity profile f. It is evident from the figure that an increase in α results in the decrease of the velocity; here boundary layer thickness increases and shear thickening is observed.
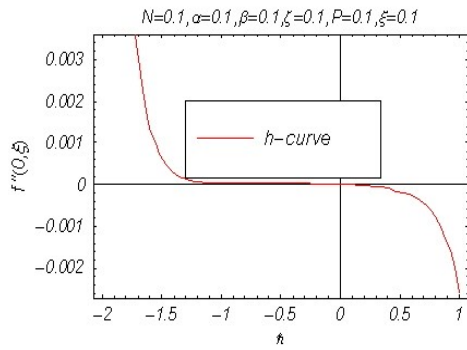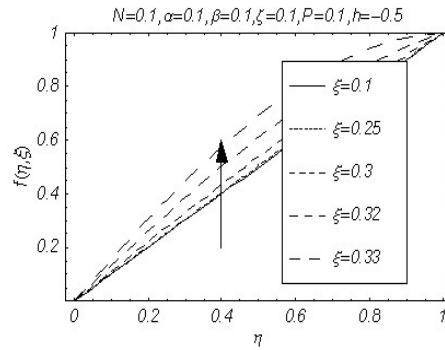
Fig.1: *ħ-curve* for *f(η, ξ)*.

Fig. 2: Influence of *ξ* on *f(η, ξ)*.
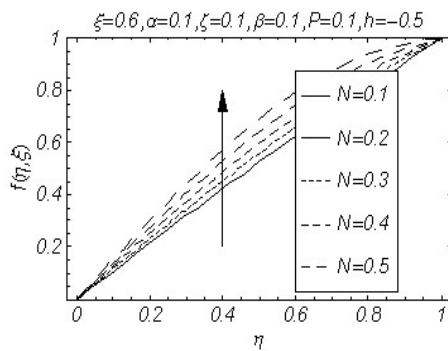
Fig. 3: Influence of *N* on *f(η, ξ)*.
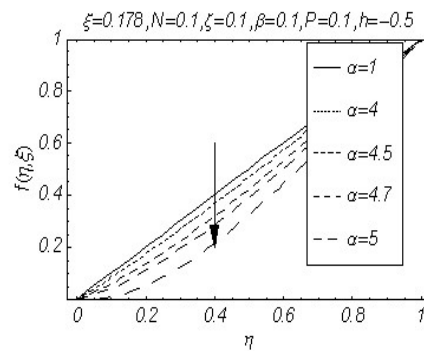

Fig. 4: Influence of *α* on *f(η, ξ)*.

In Fig. 5 and Fig. 6 the velocity distribution is presented for the various values of third-grade parameters *β* and *ζ*. It is observed that the velocity increases by increasing the influence of *β* and decreases by increasing the influence of *ζ*.
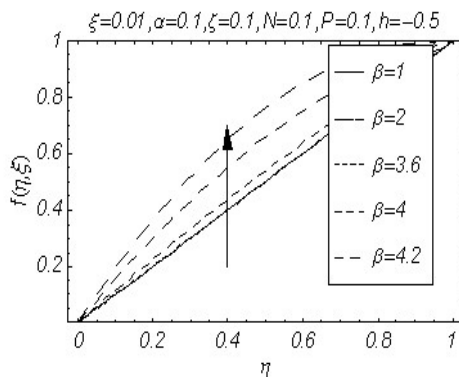

Fig. 5: Influence of *β* on f (η, ξ).
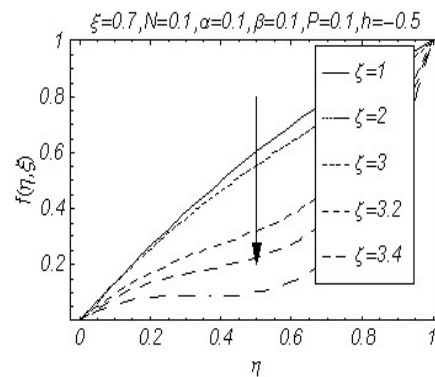
Fig. 6: Influence of *ζ* on *f (η, ξ)*.
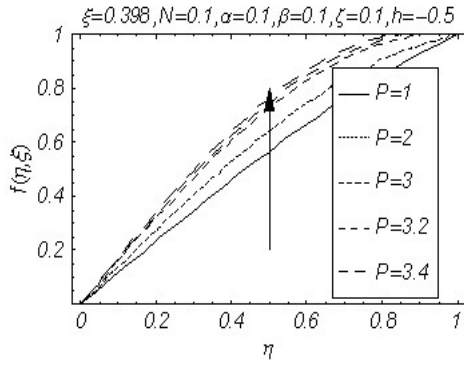
Fig. 7: Influence of *P* on *f* (*η, ζ*).
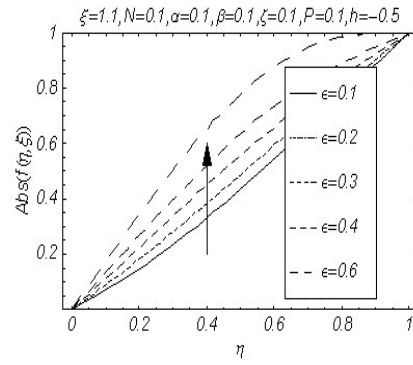
Fig. 8: Influence of *ε* on Abs *{f* (*η, ξ*)*}*.

Figure 7 depicts the variation of the pressure number on the velocity. It is observed that the velocity increases with an increase in *P*, which is consistent with what is expected. Due to the Hall current velocity field becoming complex, the absolute value of the velocity profile is plotted in Fig. 8. It is observed that with an increase in the Hall parameter *ε*, the absolute value of the velocity increases and boundary layer thickness decreases.

It is observed from Table 1 that for fixed values of *α, β, ζ, ħ, P* and *ε* with increase in Hartmann number *N*, the absolute value of the skin friction co-efficient $R_e \times C_f$ given in equation (16) increases for all values of the time *ξ*. Also with an increase in the Hall current *ε*, the absolute value of the skin friction co-efficient decreases for all values of the time *ξ*. For fixed values of *N, ħ, α, β, ζ, P* and *ε* with increase in dimensionless time *ξ* the absolute value of the skin friction co-efficient decreases at all points. Table 2 displays the variation of the skin friction co-efficient with the pressure number *P*. It is observed from Table 2 that increase in the pressure leads to an increase in the shear stress at the wall at all points which is consistent with our expectation.

Table 1: Absolute values of the skin friction co-efficient $R_e \times C_f$ with
$\alpha = 0.1, \beta = 0.1, \zeta = 0.1, P = 0.1, \hbar = 0.5.$

ξ	ε = 0.1 N = 0.1	ε = 0.1 N = 0.3	ε = 0.1 N = 0.5	N = 0.1 ε = 0.2	N = 0.1 ε = 0.3	N = 0.1 ε = 0.5
0.2	1.98649	1.99305	1.99961	1.98639	1.98625	1.98586
0.4	1.89823	1.92579	1.95333	1.89783	1.89722	1.89559
0.6	1.80866	1.87047	1.93679	1.80777	1.80641	1.80276
0.8	1.74278	1.84279	1.93615	1.74147	1.73924	1.73350
1	1.74219	1.84016	1.93205	1.74139	1.73920	1.73325

Table 2: Absolute values of the skin friction co-efficient $R_e \times C_f$ with
$\alpha = 0.1, \beta = 0.1, \zeta = 0.1, \xi = 0.1, \hbar = 0.5.$

P	ε = 0.1 N = 0.1	ε = 0.1 N = 0.3	ε = 0.1 N = 0.5	N = 0.1 ε = 0.2	N = 0.1 ε = 0.3	N = 0.1 ε = 0.5
0.2	2.02849	2.03009	2.03168	2.02847	2.02844	2.02834
0.4	2.03078	2.03237	2.03396	2.03076	2.03072	2.03063
0.6	2.03306	2.03465	2.03624	2.03304	2.03300	2.03291
0.8	2.03535	2.03694	2.03853	2.03532	2.03529	2.03519
1.0	2.03763	2.03922	2.04081	2.03761	2.03757	2.03748

4. CONCLUSION

The Couette flow between two parallel plates filled with MHD unsteady third-grade fluid is studied analytically. The effects of the pressure and Hall current are also incorporated. A non-linear third-grade model for the fluid is used. The model is invoked into the governing equations and the resulting one dimensional equation for unsteady MHD flow is derived. This equation is solved by HAM in general to study the sensitivity of the flow to the parameters that are used in the third-grade model. The various dimensionless parameters seem to affect the velocity a lot. The velocity profile and local skin friction co-efficient are greatly influenced by the Hall parameter, pressure and Hartmann numbers.

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