

THE CAUCHY PROBLEM FOR THE SYSTEM OF EQUATIONS OF THERMOELASTICITY IN E^n

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ABSTRACT: In this paper, we consider the problem of analytical continuation of solutions to the system of equations of thermoelasticity in a bounded domain. That is, we make a detailed analysis of the Cauchy problem regarding the values of thermoelasticity in bounded regions and the associated values of their strains on a part of the boundary of this domain.

ABSTRAK: Di dalam kajian ini, kami menyelidiki masalah keselanjaran analitik bagi penyelesaian-penyelesaian terhadap sistem persamaan-persamaan termoelelastik di dalam domain bersempadan berdasarkan nilai-nilainya dan nilai tegasannya bagi sebahagian daripada sempadan domain tersebut, iaitu kami mengkaji masalah Cauchy.

KEYWORDS: *Cauchy problem; system theory of elasticity; elliptic system; ill-posed problem; Carleman matrix; regularization*

1. INTRODUCTION

In this paper, we consider the problem of analytical continuation of the solution of the system equations of the thermoelasticity in spacious bounded domain from its values and values of its strains on part of the boundary of this domain, i.e., we study the Cauchy problem. Since, in many actual problems, either a part of the boundary is inaccessible for measurement of displacement and tensions or only some integral characteristics are available. Therefore, it is necessary to consider the problem of continuation for the solution of elasticity system of equations to the domain by values of the solutions and normal derivatives in the part of boundary of domain.

The system of equations of thermoelasticity is elliptic. Therefore, the Cauchy problem for this system is ill-posed. For ill-posed problems, one does not prove the existence theorem: the existence is assumed a priori. Moreover, the solution is assumed to belong to some given subset of the function space, usually a compact one [1]. The uniqueness of the solution follows from the general Holmgren theorem [2]. On establishing uniqueness in the article studio of ill-posed problems, one comes across important questions concerning the derivation of estimates of conditional stability and the construction of regularizing operators. Our aim is to construct an approximate solution using the Carleman function method.

Let $x = (x_1, \dots, x_n)$ and $y = (y_1, \dots, y_n)$ be points of the n -dimensional Euclidean space E^n , D a bounded simply connected domain in E^n , with piecewise-smooth boundary consisting of a piece Σ of the plane $y_n = 0$ and a smooth surface S lying in the half-space $y_n > 0$.

Suppose $U(x) = (u_1(x), \dots, u_n(x), u_{n+1}(x))$ is a vector function which satisfies the following system of equations of thermoelasticity in D [3]:

$$B(\partial_x, \omega)U(x) = 0, \tag{1}$$

where $B(\partial_x, \omega) = \|B_{kj}(\partial_x, \omega)\|_{(n+1) \times (n+1)}$,

and $B_{kj}(\partial_x, \omega) = \delta_{kj}(\mu\Delta + \rho\omega^2) + (\lambda + \mu)\frac{\partial^2}{\partial x_k \partial x_j}$, $k, j = 1, \dots, n$,

$$B_{k(n+1)}(\partial_x, \omega) = -\gamma \frac{\partial}{\partial x_{(n+1)}}, \quad k = 1, \dots, n,$$

$$B_{(n+1)j}(\partial_x, \omega) = i\omega\eta \frac{\partial}{\partial x_j}, \quad j = 1, \dots, n,$$

$$B_{(n+1)(n+1)}(\partial_x, \omega) = \Delta + \frac{i\omega}{\theta},$$

δ_{ij} is the Kronecker delta, ω is the frequency of oscillation and $\lambda, \mu, \rho, \theta$ its coefficients which characterize the medium, satisfying the conditions

$$\mu > 0, 3\lambda + 2\mu > 0, \rho > 0, \theta > 0, \frac{\gamma}{\eta} > 0.$$

The system (1) may be written in the following manner:

$$\begin{cases} \mu\Delta u + (\lambda + \mu)\text{grad div}u - \gamma\text{grad}v + \rho\omega^2 u = 0 \\ \Delta v + \frac{i\omega}{\theta}v + i\omega\eta\text{div}u = 0, \end{cases}$$

where $U(x) = (u(x), v(x))$.

This system is elliptic, since, its characteristic matrix is

$$\chi(\xi) = \begin{vmatrix} (\lambda + \mu)\xi_1^2 + \mu\sum_{i=1}^n \xi_i^2 & (\alpha + \mu)\xi_1\xi_2 & \Lambda & (\alpha + \mu)\xi_1\xi_n & 0 \\ (\alpha + \mu)\xi_2\xi_1 & (\lambda + \mu)\xi_2^2 + \mu\sum_{i=1}^n \xi_i^2 & \Lambda & (\alpha + \mu)\xi_2\xi_n & 0 \\ \Lambda & \Lambda & \Lambda & \Lambda & \Lambda \\ 0 & 0 & \Lambda & 0 & 1 \end{vmatrix},$$

and for arbitrary $\xi = (\xi_1, \dots, \xi_n)$ with real components satisfying the conditions

$$\sum_{i=1}^n \xi_i^2 = 1, \text{ we have}$$

$$\det\chi(\xi) = \mu^2(\lambda + \mu) \neq 0.$$

Statement of the problem. Find a regular solution U of the system (1) in the domain D by using its Cauchy data on the surface S :

$$U(y) = f(y), \quad R(\partial_y, \nu(y))U(y) = g(y), \quad y \in S, \tag{2}$$

where $R(\partial_y, \nu(y))$ is the stress operator, i.e.,

$$R(\partial_y, \nu(y)) = \left\| R_{kj}(\partial_y, \nu(y), \gamma) \right\|_{(n+1) \times (n+1)} = \left\| \begin{array}{cc} & -\gamma \nu_1 \\ T & -\gamma \nu_2 \\ \dots & \dots \\ \dots & \dots \\ 0 & 0 \dots \dots \frac{\partial}{\partial \nu} \end{array} \right\|,$$

$$T = T(\partial_y, \nu) = \left\| T_{kj}(\partial_y, \nu) \right\|_{n \times n},$$

$$T_{kj}(\partial_y, \nu) = \lambda \nu_k \frac{\partial}{\partial y_j} + \mu \nu_j(y) \frac{\partial}{\partial y_k} + (\mu + \lambda) \delta_{kj} \frac{\partial}{\partial \nu(y)}, \quad k, j = 1, \dots, n,$$

$\nu(y) = (\nu_1(y), \dots, \nu_n(y))$ is the unit outward normal vector on ∂D at a point y , $f = (f_1, K, f_{n+1})$, $g = (g_1, K, g_{n+1})$ are given continuous vector functions on S .

2. CONSTRUCTION OF THE CARLEMAN MATRIX AND APPROXIMATE SOLUTION FOR THE CAP TYPE DOMAIN

It is well known that any regular solution $U(x)$ of the system (1) is specified by the formula

$$2U(x) = \int_{\partial D} (\Psi(x-y, \omega) \{R(\partial_y, \nu(y))U(y)\} - \{ \tilde{R}(\partial_y, \nu(y)) \tilde{\Psi}(y-x, \omega) \}^* U(y)) ds_y, \quad x \in D, \tag{3}$$

where the symbol $*$ – means the operation of transposition, Ψ is the matrix of the fundamental solutions for the system of equations of steady-state oscillations of thermoelasticity: given by

$$\Psi(x, \omega) = \left\| \Psi_{kj}(x, \omega) \right\|_{(n+1) \times (n+1)},$$

$$\Psi_{kj}(x, \omega) = \sum_{l=1}^3 [(1 - \delta_{k(n+1)})(1 - \delta_{j(n+1)}) \left(\frac{\delta_{kl}}{2\pi\mu} \delta_{nl} - \alpha_l \frac{\partial^2}{\partial x_k \partial x_j} \right) + \beta_l \left(i\omega\eta \delta_{k(n+1)} (1 - \delta_{j(n+1)}) \frac{\partial}{\partial x_j} - \gamma \delta_{j(n+1)} (1 - \delta_{k(n+1)}) \frac{\partial}{\partial x_k} \right) + \delta_{k(n+1)} \delta_{j(n+1)} \gamma_l] \frac{\exp(i\lambda_l |x|)}{|x|},$$

$$\alpha_l = \frac{(-1)^l (1 - i\omega\theta^{-1} \lambda^{-1} (\delta_{1l} + \delta_{2l}))}{2\pi(\lambda + 2\mu)(\lambda_2^2 - \lambda_1^2)} - \frac{\delta_{3l}}{2\pi\rho\omega^2}, \quad l = 1, 2, 3; \quad \sum_{l=1}^3 \alpha_l = 0,$$

$$\beta_l = \frac{(-1)^l (\delta_{1l} + \delta_{2l})}{2\pi(\lambda + 2\mu)(\lambda_2^2 - \lambda_1^2)}, \quad l = 1, 2, 3; \quad \sum_{l=1}^3 \beta_l = 0,$$

$$\gamma_l = \frac{(-1)^l (\lambda_2^2 - k_1^2)(\delta_{1l} + \delta_{2l})}{2\pi(\lambda_2^2 - \lambda_1^2)}, \quad l = 1, 2, 3; \quad \sum_{l=1}^3 \gamma_l = 0, \quad k_1^2 = \rho\omega^2 (\lambda + 2\mu)^{-1},$$

$$\tilde{\Psi}(x, \omega) = \Pi \tilde{\Psi}_{kj}(x, \omega) \Pi_{(n+1) \times (n+1)}, \tilde{\Psi}_{kj}(x, \omega) = \Psi_{kj}(-x, \omega),$$

$$\tilde{R}(\partial_y, \nu(y)) = \left\| \begin{array}{cccc} & & -i\omega\nu_1 & \\ & T & -i\omega\nu_2 & \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & -i\omega\nu_n \\ 0 & 0 & \dots & \dots \frac{\partial}{\partial \nu} \end{array} \right\|.$$

Definition. By the Carleman matrix of the problem (1),(2) we mean an $(n+1) \times (n+1)$ matrix $\Pi(y, x, \omega, \tau)$ depending on the two points y, x and a positive numerical number parameter τ satisfying the following two conditions:

$$1) \Pi(y, x, \omega, \tau) = \Psi(x - y, \omega) + G(y, x, \tau),$$

where the matrix $G(y, x, \tau)$ satisfies system (1) with respect to the variable y on D , and $\Psi(y, x)$ is a matrix of the fundamental solutions of system (1);

$$2) \int_{\partial D \setminus S} (|\Pi(y, x, \omega, \tau)| + |R(\partial_y, \nu)\Pi(y, x, \omega, \tau)|) ds_y \leq \varepsilon(\tau),$$

where $\varepsilon(\tau) \rightarrow 0$, as $\tau \rightarrow \infty$; here $|\Pi|$ is the Euclidean norm of the matrix

$$\Pi = \|\Pi_{ij}\|_{(n+1) \times (n+1)}, \text{ i.e., } |\Pi| = \left(\sum_{i,j=1}^{n+1} \Pi_{ij}^2 \right)^{\frac{1}{2}}. \text{ In particular, } |U| = \left(\sum_{m=1}^{n+1} u_m^2 \right)^{\frac{1}{2}}.$$

From the definition of Carleman matrix it follows that.

Theorem 1. Any regular solution $U(x)$ of system (1) in the domain D is specified by the formula

$$2U(x) = \int_{\partial D} (\Pi(y, x, \omega, \tau) \{R(\partial_y, \nu)U(y)\} - \{\tilde{R}(\partial_y, \nu)\Pi(y, x, \omega, \tau)\}^* U(y)) ds_y, \quad x \in D, \tag{4}$$

where $\Pi(y, x, \omega, \tau)$ is the Carleman matrix.

Using this matrix, one can easily conclude the estimated stability of solution of the problem (1), (2) and also indicate effective method decision this problem as in [4 - 6].

With a view to construct an approximate solution of the problem (1), (2) we construct the following matrix:

$$\Pi(y, x, \omega) = \|\Pi_{kj}(y, x, \omega)\|_{(n+1) \times (n+1)}, \tag{5}$$

$$\begin{aligned} \Pi_{kj}(y, x, \omega) = & \sum_{l=1}^3 [(1 - \delta_{k(n+1)})(1 - \delta_{j(n+1)}) \left(\frac{\delta_{kj}}{2\pi\mu} \delta_{nl} - \alpha_l \frac{\partial^2}{\partial x_k \partial x_j} \right) + \\ & + \beta_l \left(i\omega\eta \delta_{k(n+1)} (1 - \delta_{j(n+1)}) \frac{\partial}{\partial x_j} - \gamma \delta_{j(n+1)} (1 - \delta_{k(n+1)}) \frac{\partial}{\partial x_k} \right) + \delta_{k(n+1)} \delta_{j(n+1)} \gamma_l] \Phi(y, x, k_l), \end{aligned}$$

where

$$C_n K(x_n) \Phi(y, x, k) = \int_0^\infty \text{Im} \left[\frac{K(i\sqrt{u^2 + s} + y_n)}{i\sqrt{u^2 + s} + y_n - x_n} \right] \frac{\psi(ku) du}{\sqrt{u^2 + s}}, \tag{6}$$

$$\psi(ku) = \begin{cases} uJ_0(ku), & n = 2m, \quad m \geq 1, \\ \cos ku, & n = 2m + 1, \quad m \geq 1, \end{cases} \quad J_0(u) \text{-Bessel's function of order zero,}$$

$$s = (y_1 - x_1)^2 + \dots + (y_{n-1} - x_{n-1})^2$$

and

$$C_2 = 2\pi, C_n = \begin{cases} (-1)^m \cdot 2^{-n} (n-2)\pi\omega_n (m-2)!, & n = 2m \\ (-1)^m \cdot 2^{-n} (n-2)\pi\omega_n (m-1)!, & n = 2m + 1. \end{cases}$$

$K(\omega), \omega = u + iv$ (u, v are real), is an entire function taking real values on the real axis and satisfying the conditions:

$$K(u) \neq \infty \text{ for } |u| < \infty, \quad K(u) \neq 0,$$

$$\sup_{v \geq 1} |\exp v| \operatorname{Im} |K^{(p)}(\omega)| = M(p, u) < \infty, \quad p = 0, \dots, m, \quad u \in R^1.$$

The following theorem was proved in [7].

Lemma 1. For function $\Phi(y, x, k)$ the following formula is valid

$$C_n \Phi(y, x, k) = \varphi_n(ikr) + g_n(y, x, k), \quad r = |y - x|,$$

where φ_n - are fundamental solutions of the Helmholtz equation, $g_n(y, x, k)$ is a regular function that is defined for all y and x satisfies the Helmholtz equation: $\Delta(\partial_y)g_n - k^2 g_n = 0$.

In (6) we put $K(\omega) = \exp(\tau\omega)$. Then

$$\Phi(y, x, k) = \Phi_\tau(y - x, k),$$

$$\begin{aligned} C_n \Phi_\tau(y - x, k) &= \frac{\partial^{m-1}}{\partial s^{m-1}} \int_0^\infty \operatorname{Im} \left[\frac{\exp \tau(i\sqrt{u^2 + s} + y_n - x_n)}{i\sqrt{u^2 + s} + y_n - x_n} \right] \frac{\psi(ku) du}{\sqrt{u^2 + s}} \\ &= \exp \tau(y_n - x_n) \frac{\partial^{m-1}}{\partial s^{m-1}} \int_0^\infty \left[-\cos \tau\sqrt{u^2 + \alpha^2} + (y_n - x_n) \frac{\sin \tau\sqrt{u^2 + s}}{\sqrt{u^2 + s}} \right] \psi(ku) du, \end{aligned} \quad (7)$$

$$\Phi'_\tau(y - x, k) = \frac{\partial \Phi_\tau}{\partial \tau}.$$

$$C_n \Phi'_\tau(y - x, k) = \exp \tau(y_n - x_n) \frac{\partial^{m-1}}{\partial s^{m-1}} \int_0^\infty \frac{\sin \tau\sqrt{u^2 + s}}{\sqrt{u^2 + s}} \psi(ku) du,$$

$$C_n \Phi'_\tau(y - x, k) = \exp \tau(y_n - x_n) \frac{\partial^{m-1}}{\partial s^{m-1}} \psi'_\tau(k, s),$$

$$\psi'_\tau = \begin{cases} 0, & \tau < k \\ \cos \sqrt{s(\tau^2 - k^2)}, & n = 2m \\ \frac{1}{2} \pi J_0(\sqrt{s(\tau^2 - k^2)}), & \tau > k \end{cases}$$

Now, in formulas (5) and (6) we set $\Phi(y, x, k) = \Phi_\tau(y - x, k)$ and construct the matrix $\Pi(y, x, \omega) = \Pi(y, x, \omega, \tau)$

From Lemma 1 we obtain,

Lemma 2. The matrix $\Pi(y, x, \omega, \tau)$ given by (5) and (6) is Carleman's matrix for problem (1), (2).

Indeed by (5), (6) and Lemma 1 we have

$$\Pi(y, x, \omega, \tau) = \Psi(y, x, \omega) + G(y, x, \tau),$$

where

$$G(y, x, \tau) = \|G_{kj}(y, x, \tau)\|,$$

$$G_{kj}(y, x, \tau) = \sum_{l=1}^3 [(1 - \delta_{k(n+1)})(1 - \delta_{j(n+1)}) \left(\frac{\delta_{kj}}{2\pi\mu} \delta_{nl} - \alpha_l \frac{\partial^2}{\partial x_k \partial x_j} \right) + \beta_l \left(i\omega\eta\delta_{k(n+1)}(1 - \delta_{j(n+1)}) \frac{\partial}{\partial x_j} - \gamma\delta_{j(n+1)}(1 - \delta_{k(n+1)}) \frac{\partial}{\partial x_k} \right) + \delta_{k(n+1)}\delta_{j(n+1)}\gamma_l] g_n(y, x, k_l, \tau), \quad k, j = 1, \dots, n + 1.$$

By a straightforward calculation, we can verify that the matrix $G(y, x, \tau)$ satisfies the system (1) with respect to the variable y everywhere in D . By using (5), (6) and (7) we obtain

$$\int_{\partial D \setminus S} (|\Pi(y, x, \omega, \tau)| + |R(\partial_y, \nu)\Pi(y, x, \omega, \tau)|) ds_y \leq C_1(x)\tau^m \exp(-\tau x_n), \quad (8)$$

Where $C_1(x)$ is a bounded function inside of D .

Let us set

$$2U_\tau(x) = \int_S (\Pi(y, x, \omega, \tau)\{R(\partial_y, \nu)U(y)\} - \{\tilde{R}(\partial_y, \nu)\Pi(y, x, \omega, \tau)\}^* U(y)) ds_y. \quad (9)$$

The following theorem holds.

Theorem 2. Let $U(x)$ be a regular solution of the system (1) in D such that

$$|U(y)| + |R(\partial_y, \nu)U(y)| \leq M, \quad y \in \partial D \setminus S. \quad (10)$$

Then for $\tau \geq 1$ the following estimate is valid:

$$|U(y) - U_\tau(y)| \leq MC_2(x)\tau^m \exp(-\tau x_n).$$

By formulas (4) and (9) we have

$$2|U(x) - U_\tau(x)| = \int_{\partial D \setminus S} (\Pi(y, x, \omega, \tau)\{R(\partial_y, \nu)U(y)\} - \{\tilde{R}(\partial_y, \nu)\Pi(y, x, \omega, \tau)\}^* U(y)) ds_y.$$

Now on the basis of (8) and (10) we obtain the required estimate.

Next we write out a result that allows us to calculate $U(x)$ approximately if, instead of $U(y)$ and $R(\partial_y, \nu)U(y)$, its continuous approximations $f_\delta(y)$ and $g_\delta(y)$ are given on the surface S :

$$\max_S |f(y) - f_\delta(y)| + \max_S |R(\partial_y, \nu)U(y) - g_\delta(y)| \leq \delta, \quad 0 < \delta < 1. \quad (11)$$

We define a function $U_{\tau\delta}(x)$ by setting

$$2U_{\tau\delta}(x) = \int_S (\Pi(y, x, \omega, \tau)g_\delta(y) - \{\tilde{R}(\partial_y, \nu)\Pi(y, x, \omega, \tau)\}^* f_\delta(y)) ds_y, \quad (12)$$

where

$$\tau = \frac{1}{x_n^0} \ln \frac{M}{\delta}, \quad x_n^0 = \max_D x_n, \quad x_n > 0.$$

Then the following theorem holds:

Theorem 3. Let $U(x)$ be a regular solution of the system (1) in D satisfying the condition (10). Then the following estimate is valid:

$$|U(x) - U_{\tau\delta}(x)| \leq C_3(x) \delta^{x_n^0} \left(\ln \frac{M}{\delta} \right)^m, \quad x \in D.$$

From all of the above results we immediately obtain a stability estimate.

Theorem 4. Let $U(x)$ be a regular solution of the system (1) in D satisfying the conditions:

$$|U(y)| + |R(\partial_y, \nu)U(y)| \leq M, \quad y \in \partial D \setminus S$$

and

$$|U(y)| + |R(\partial_y, \nu)U(y)| \leq \delta, \quad 0 < \delta < 1, \quad y \in S.$$

Then

$$|U(x)| \leq C_4(x) \delta^{x_n^0} \left(\ln \frac{M}{\delta} \right)^m,$$

where $C_4(x) = \tilde{C} \int_{\partial D} \frac{1}{|x-y|^n} ds_y$, \tilde{C} is a constant depending on λ, μ, ω .

Corollary 1. The limits

$$\lim_{\tau \rightarrow \infty} U_\tau(x) = U(x), \quad \lim_{\delta \rightarrow 0} U_{\tau\delta}(x) = U(x)$$

hold uniformly on each compact subset of D .

3. REGULARIZATION OF SOLUTION OF THE PROBLEM (1), (2) FOR A CONE TYPE DOMAIN

Let $x = (x_1, K, x_n)$ and $y = (y_1, K, y_n)$ be points in E^n , D_ρ be a bounded simply connected domain in E^n whose boundary consists of a cone surface

$$\Sigma: \alpha_1 = \tau_\rho y_n, \alpha_1^2 = y_1^2 + K + y_{n-1}^2, \tau_\rho = \operatorname{tg} \frac{\pi}{2\rho}, y_n > 0, \rho > 1$$

and a smooth surface S lying in the cone. Assume $x_0 = (0, \dots, 0, x_n) \in D_\rho$.

We construct Carleman matrix. In formula (5), (6) we set

$$K(\omega) = E_\rho [\tau(\omega - x_n)], \quad \tau > 0, \rho > 1.$$

Then

$$\Phi(y, x, k) = \Phi_\tau(y - x, k), \quad k > 0$$

$$C_n \Phi_\tau(y - x, k) = \frac{\partial^{m-1}}{\partial s^{m-1}} \int_0^\infty \text{Im} \left[\frac{E_\rho(\tau(i\sqrt{u^2 + s} + y_n - x_n))}{i\sqrt{u^2 + s} + y_n - x_n} \right] \frac{\psi(ku) du}{\sqrt{u^2 + s}} \quad (13)$$

$$\Phi'_\tau(y - x, k) = \frac{\partial \Phi_\tau}{\partial \tau}.$$

$$C_n \Phi'_\tau(y - x, k) = \frac{\partial^{m-1}}{\partial s^{m-1}} \int_0^\infty \text{Im} \left\{ E'_\rho \left[\tau(i\sqrt{u^2 + s} + y_n - x_n) \right] \right\} \frac{\psi(ku) du}{\sqrt{u^2 + s}},$$

Where $E_\rho(w)$ is Mittag-Löffer's entire function [8]. For the functions $\Phi_\tau(y - x, k)$ hold Lemma 1 and Lemma 2.

Further, we may show similar estimate for $U_\tau(x)$ and $U_{\omega}(x)$ (in cone case) defined in (9) and (12), as Theorems 1, 2, 3, and 4.

For the simplicity let us consider $n = 3$, since the other cases are considered analogously.

Suppose that D_ρ is a bounded simple connected domain in E^3 with boundary consisting of part Σ of the surface of the cone

$$y_1^2 + y_2^2 = \tau_\rho y_3^2, \quad \tau_\rho = tg \frac{\pi}{2\rho}, \quad \rho > 1, \quad y_3 > 0,$$

and of a smooth part of the surface S lay inside the cone. Assume $x_0 = (0, 0, x_3) \in D_\rho$.

We construct Carleman's matrix. In formulas (5), (6) we take

$$\Phi_\tau(y, x, k) = \frac{1}{4\pi^2 E_\rho(\tau^\rho x_3)} \int_0^\infty \text{Im} \frac{E_\rho(\tau^\rho w)}{i\sqrt{u^2 + s} + y_3 - x_3} \frac{\cos kudu}{\sqrt{u^2 + s}}, \quad (14)$$

where $w = i\sqrt{u^2 + s} + y_3$. For the functions $\Phi_\tau(y, x, k)$ holds Lemma 1.

It follows from the properties of $E_\rho(w)$ that for $y \in \Sigma$ and $0 < u < \infty$ the function $\Phi_\tau(y, x, k)$ defined by (13) its gradient and the second partial derivatives

$$\frac{\partial^2 \Phi_\tau(y, x, k)}{\partial y_k \partial y_j}, \quad k, j = 1, 2, 3,$$

tends to zero as $\tau \rightarrow \infty$, for a fixed $x \in D_\rho$.

Then from (5) we find that the matrix $\Pi(y, x, \omega, \tau)$ and its stresses $R(\partial_y, \nu)\Pi(y, x, \omega, \tau)$ also converge to zero as $\tau \rightarrow \infty$ for all $y \in \Sigma$, i.e., $\Pi(y, x, \omega, \tau)$ is the Carleman matrix for the domain D_ρ and the part Σ of the boundary.

If $U(x)$ is a regular solution of the system (1) then the following integral formula

$$2U(x) = \int_{\partial D_\rho} (\Pi(y, x, \omega, \tau) \{R(\partial_y, \nu)U(y)\} - \{\tilde{R}(\partial_y, \nu)\Pi(y, x, \omega, \tau)\}^* U(y)) ds_y.$$

holds. For $x \in D_\rho$ we denote by $U_\tau(x)$ the following:

$$2U_\tau(x) = \int_S (\Pi(y, x, \omega, \tau) \{R(\partial_y, \nu)U(y)\} - \{\tilde{R}(\partial_y, \nu)\Pi(y, x, \omega, \tau)\}^* U(y)) ds_y. \quad (15)$$

Then the following theorem holds.

Theorem 5. Let $U(x)$ be a regular solution of the system (1) in D_ρ such that

$$|U(y)| + |R(\partial_y, \nu)U(y)| \leq M, \quad y \in \Sigma. \quad (16)$$

Then for $\tau \geq 1$ the following estimate is valid:

$$|U(x_0) - U_\tau(x_0)| \leq MC_\rho(x_0)\tau^3 \exp(-\tau x_3^\rho),$$

where $x_0 = (0, 0, x_3) \in D_\rho, \quad x_3 > 0,$

$$C_\rho(x_0) = C_\rho \int_\Sigma \frac{1}{r_0^3} ds_y, \quad r_0 = |y - x_0|, \quad C_\rho - \text{constant}.$$

Let us take continuous approximations $f_\delta(y)$ and $g_\delta(y)$ of $U(y)$ and $R(\partial_y, \nu)U(y)$, respectively, i. e.,

$$\max_S |U(y) - f_\delta(y)| + \max_S |R(\partial_y, \nu)U(y) - g_\delta(y)| \leq \delta, \quad 0 < \delta < 1$$

and define the following function

$$2U_{\tau\delta}(x) = \int_S (\Pi(y, x, \omega, \tau)g_\delta(y) - \{\tilde{R}(\partial_y, \nu)\Pi(y, x, \omega, \tau)\}^* f_\delta(y)) ds_y,$$

Then the following theorem holds.

Theorem 6. Let $U(x)$ be a regular solution of the system (1) in the domain D_ρ satisfying the condition (16), then

$$|U(x_0) - U_{\tau\delta}(x_0)| \leq C_\rho(x_0)\delta^q \left(\ln \frac{M}{\delta}\right)^3,$$

where $\tau = (\tau_\rho R)^{-\rho} \ln \frac{M}{\delta}, \quad R^\rho = \max_S \operatorname{Re}(i\sqrt{s} + y_3)^\rho,$

$$q = \left(\frac{x_3}{R}\right)^\rho, \quad C_\rho(x_0) = C_\rho \int_\Sigma \left[\frac{1}{r_0^3} + \frac{1}{r_0^4}\right] ds_y.$$

The theorem is proved analogously as Theorems 3 and 4.

Corollary 2. The limits

$$\lim_{\tau \rightarrow \infty} U_\tau(x) = U(x), \quad \lim_{\delta \rightarrow 0} U_{\tau\delta}(x) = U(x)$$

hold uniformly on each compact subset of D_ρ .

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