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Integration of Neutrosophic Methods into Adaptive Control of Nonlinear Systems Using Neuro-Fuzzy Networks With B-Spline Functions

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ABSTRACT: This paper addresses the problem of adaptive control of nonlinear dynamic systems operating under parametric uncertainty, external disturbances, and partial or contradictory information about the system state – conditions under which classical linear Model Reference Adaptive Control (MRAC) and conventional neuro-fuzzy controllers exhibit degraded performance, slow adaptation, and oscillatory behavior. To overcome these limitations, a novel Neuro-Neutrosophic Model Reference Adaptive Controller (NN-MRAC) is proposed, implemented using a neutrosophic neuro-fuzzy network with B-spline basis functions. The key innovation of the proposed approach lies in integrating neutrosophic logic into the adaptive control architecture by explicitly using a three-component uncertainty representation – truth, indeterminacy, and falsity – which enables robust control synthesis in the presence of incomplete, noisy, and conflicting data. In contrast to traditional neuro-fuzzy controllers, the proposed NN-MRAC combines localized B-spline approximation with neutrosophic weighting of local models and an adaptive decomposition into lower-dimensional submodels, effectively mitigating the curse of dimensionality and reducing computational complexity. Comparative simulation studies with a classical linear MRAC demonstrate that the proposed controller reduces the mean-square tracking error by approximately 59%, decreases overshoot by more than 3 times, shortens the transient response time by nearly 1.8 times, and lowers control energy consumption by about 18%. The results confirm that the proposed neuro-neutrosophic MRAC ensures stable, smooth, and energy-efficient control in the presence of noise and deep uncertainty, making it a promising solution for intelligent control of complex nonlinear systems.

ABSTRAK: Kajian ini mencadangkan masalah kawalan adaptif bagi sistem dinamik bukan linear yang beroperasi pada ketidakpastian parameter, gangguan luaran, serta maklumat separa atau bercanggah mengenai keadaan sistem, di mana kaedah linear klasik Model Kawalan Suai Rujukan (MRAC) dan kawalan konvensional neural-fuzi menunjukkan prestasi terhad, kadar penyesuaian perlahan, dan tingkah laku berayun. Bagi mengatasi kekangan ini, satu Model Kawalan Suai Rujukan Neuro-Neutrosofik (NN-MRAC) yang baharu dicadangkan, dilaksanakan berasaskan rangkaian neural-fuzi neutrosofik dengan fungsi asas alur-B. Inovasi utama pendekatan ini terletak pada pengintegrasian logik neutrosofik ke dalam seni bina kawalan suai melalui penggunaan perwakilan terkecuali pada tiga komponen ketidakpastian – kebenaran, ketidakpastian, dan kepalsuan – membolehkan sintesis kawalan yang teguh dalam keadaan data tidak lengkap, bising, dan bercanggah. Berbeza dengan pengawal neural-fuzi tradisional, NN-MRAC yang dicadangkan menggabungkan penghampiran setempat berasaskan alur-B dengan pemberat neutrosofik bagi model setempat serta penguraian suai kepada submodel berdimensi rendah, sekaligus mengurangkan kerumitan pengiraan dan mengatasi masalah “kutukan dimensi”. Kajian simulasi perbandingan dengan MRAC linear klasik menunjukkan bahawa cadangan kawalan mencapai pengurangan ralat min kuasa dua (MSE) kira-kira 59%, penurunan lebihan (overshoot) lebih daripada tiga kali ganda, pemendekan masa tindak balas sementara hampir 1.8 kali, serta pengurangan penggunaan tenaga kawalan sekitar 18%. Keputusan ini mengesahkan bahawa MRAC neuro-neutrosofik yang dicadangkan mampu memastikan kawalan stabil, licin, dan cekap tenaga di bawah keadaan hingar dan ketidakpastian mendalam, menjadikannya satu penyelesaian berpotensi pada sistem kawalan pintar bukan linear yang kompleks.

KEYWORDS: *Adaptive Control, Neutrosophic Logic, MRAC, Neuro-Fuzzy Network, B-Spline.*

1. INTRODUCTION

Modern dynamic systems operate under conditions of high uncertainty, nonlinearity, and variability of external factors, which significantly complicates the synthesis of efficient control systems. Traditional methods of linear adaptive control, including classical models such as MRAC (Model Reference Adaptive Control) [1, 2], demonstrate good performance only within a narrow range of operating conditions where the object can be approximated by a linear model. However, under the influence of unpredictable disturbances, stochastic fluctuations, and parametric deviations, their effectiveness sharply decreases. In such circumstances, it becomes necessary to develop intelligent systems capable of self-learning, self-organization, and real-time adaptation. The significance of this problem is particularly pronounced in application domains such as electric power systems, robotics, autonomous and cyber-physical systems, where nonlinear dynamics, parameter drift, sensor noise, and incomplete state information are intrinsic characteristics of the operating environment. In such systems, insufficient robustness of adaptive controllers may lead not only to performance degradation but also to instability, increased energy consumption, and reduced operational safety. Therefore, the development of adaptive control strategies capable of explicitly handling uncertainty and partial state awareness represents an important and practically relevant scientific challenge.

Among various areas of intelligent control, neuro-fuzzy systems hold a special place, as they combine the heuristic principles of fuzzy logic with the universal approximating capabilities of neural networks. These hybrid structures allow modeling nonlinear dependencies without requiring an exact mathematical description of the object. An important element in such systems is the selection of basis functions, which determine the smoothness and stability of the approximation. In this context, B-spline functions possess several advantages, including localized approximation, low computational complexity, and the ability to adaptively adjust the function shape to data during the learning process. Recent studies have demonstrated that intelligent adaptive controllers based on ANFIS, neuro-PID structures, and deep neuro-fuzzy architectures can significantly improve control accuracy for nonlinear systems compared to classical linear MRAC. These approaches exploit data-driven learning and nonlinear approximation to compensate for modeling errors and disturbances. However, most existing methods implicitly assume that the available data are reliable and internally consistent, and uncertainty is typically modeled only through noise or bounded parameter variations. Despite their effectiveness, conventional neuro-fuzzy and neural adaptive controllers exhibit several fundamental limitations. First, classical fuzzy systems operate with a single degree of membership, which does not explicitly represent indeterminacy or contradictory information. Second, neural-network-based controllers often behave as black-box models, limiting interpretability and increasing the risk of unstable adaptation under sparse or conflicting data. Third, an increase in the number of input variables typically leads to a rapid growth in computational complexity, known as the curse of dimensionality, which restricts real-time implementation in embedded and resource-constrained systems. These limitations indicate a clear research gap in adaptive control under complex and poorly structured uncertainty conditions.

Nevertheless, even conventional neuro-fuzzy models encounter significant challenges when operating under deep uncertainty, where parts of the data are contradictory, and other parts are incomplete or unreliable. To overcome these limitations, neutrosophic logic, an extension of classical fuzzy logic proposed by F. Smarandache, has been actively developed in recent years. In this approach, each variable is described by three independent components: truth (T), indeterminacy (I), and falsity (F) [3-5]. This triad enables the accounting for ambiguous and conflicting situations characteristic of complex technological systems and ensures more accurate decision-making under incomplete information. The integration of neutrosophic principles into the MRAC controller based on B-spline neuro-fuzzy networks forms a new generation of neutrosophic neuro-fuzzy adaptive control systems characterized by high robustness to uncertainty and the ability to learn in a context-dependent manner. This architecture not only expands the system's adaptive capabilities but also enables intelligent analysis of control actions that account for fuzzy, probabilistic, and neutrosophic information.

The scientific novelty of the proposed approach lies in the development of a hybrid neuro-neutrosophic MRAC that uses B-spline functions to construct local models with varying degrees of certainty (T, I, F), thereby enabling adaptive adjustment of the network's weight coefficients during system operation. The study also proposes a submodel decomposition method that reduces computational complexity and eliminates the "curse of dimensionality." Thus, the developed concept represents a step toward creating a new generation of intelligent, adaptive controllers that combine the precision of mathematical models, the flexibility of fuzzy systems, and the robustness of neutrosophic logic. The practical application of this approach is particularly relevant for control tasks in power systems, robotic systems, autonomous complexes, and other nonlinear dynamic systems operating under uncertainty.

In contrast to existing adaptive control approaches, the proposed method introduces an explicit neutrosophic representation of uncertainty within the adaptive control loop, enabling simultaneous consideration of truth, indeterminacy, and falsity in control decision-making. The use of B-spline basis functions provides a localized and smooth approximation of nonlinear dynamics, while the proposed submodel decomposition strategy ensures scalability and real-time feasibility. As a result, the developed neuro-neutrosophic MRAC differs from conventional MRAC, ANFIS, and deep neuro-fuzzy controllers by offering improved robustness to deep uncertainty, reduced computational complexity, and enhanced interpretability of control actions.

2. RELATED WORKS

Adaptive control of nonlinear systems has been extensively studied over the past decades, with classical Model Reference Adaptive Control (MRAC) forming the foundation of many modern approaches [1, 2]. While MRAC provides guaranteed stability under certain assumptions, its performance deteriorates significantly in the presence of strong nonlinearities, unmodeled dynamics, and uncertainty in system parameters. To address these limitations, intelligent control methods based on artificial neural networks and fuzzy logic have been proposed.

Adaptive Neuro-Fuzzy Inference Systems (ANFIS) and neuro-PID controllers have demonstrated improved approximation capability and adaptability compared to linear MRAC, particularly for nonlinear plants with unknown dynamics [8, 9]. These methods exploit data-driven learning to compensate for modeling inaccuracies and external disturbances. More recently, deep neuro-fuzzy and hybrid intelligent controllers have been introduced to further enhance control performance, robustness, and convergence speed by leveraging multi-layer learning architectures and nonlinear feature extraction. Despite these advances, most existing intelligent adaptive controllers rely on classical fuzzy logic or probabilistic uncertainty modeling, in which uncertainty is represented implicitly through noise, bounded disturbances, or parameter variations. Such representations do not explicitly distinguish between reliable information, incomplete knowledge, and contradictory data. As a result, these controllers may exhibit unstable adaptation, reduced interpretability, or degraded performance when operating under deep uncertainty and partial awareness of system states.

Neutrosophic logic, introduced as a generalization of fuzzy logic, provides a more expressive framework for uncertainty modeling by representing each variable using three independent components: truth, indeterminacy, and falsity [3-5]. Several recent studies have explored neutrosophic sets and inference mechanisms in decision-making, optimization, and control-related applications [14, 15]. However, the integration of neutrosophic logic into adaptive control systems, particularly in MRAC architectures and neuro-fuzzy networks, remains limited and underexplored.

In contrast to existing approaches, the present study integrates neutrosophic logic directly into the adaptive control loop of an MRAC framework using B-spline-based neuro-fuzzy networks. Unlike conventional neuro-fuzzy and deep learning controllers, the proposed method explicitly accounts for epistemic uncertainty and contradictory information while maintaining computational efficiency and interpretability through local B-spline approximation and submodel decomposition. This positions the proposed neuro-neutrosophic MRAC as a distinct and complementary contribution to the current state of the art in intelligent adaptive control of nonlinear systems.

3. THE PROBLEM OF ADAPTIVE CONTROL OF NONLINEAR SYSTEMS

Modern nonlinear systems are characterized by high uncertainty, parameter variability, and stochastic disturbances and structural nonlinearity [6]. In such systems, traditional adaptive control methods based on classical MRAC algorithms lose their effectiveness for several reasons. First, they assume that the structure of the controlled object is known in advance and can be described by an accurate mathematical model. Second, they rely on the assumption of smooth, linear dynamics in the vicinity of the operating point, which does not hold in most real installations. Finally, traditional methods do not account for the influence of multidimensional interactions and cannot effectively handle incomplete or contradictory data [7].

The typical problems that arise when applying linear MRAC systems to complex dynamic objects can be identified as follows:

1) Structural uncertainty: the parameters of the object model change over time, and its dynamics cannot be approximated by a single linear model.
2) Nonlinear dependencies between inputs and outputs: the interaction of multiple factors (temperature, load, response time, etc.) leads to nonlinear effects that cannot be compensated for by linear control.
3) Data uncertainty: measurements are often accompanied by noise, incompleteness, or even logical inconsistency, especially when working with distributed systems.

4) Computational complexity problem: an increase in the number of input variables leads to an exponential growth in model dimensionality (the “curse of dimensionality” effect), making the learning process unstable and slow.

Attempts to extend classical MRAC methods using Artificial Neural Networks (ANNs) and Fuzzy Logic have led to the emergence of hybrid neuro-fuzzy controllers [8, 9]. However, these methods also retain several limitations: first-type fuzzy systems are sensitive to noise and do not account for uncertainty within the membership functions themselves; neural networks, in turn, require large volumes of training data and often suffer from overfitting. As a result, there is a need for a more universal approach that can combine the advantages of both methods while maintaining interpretability and robustness to uncertainty.

At this point, the neutrosophic theory of control, grounded in neutrosophic logic, comes to the forefront [10, 11]. Unlike classical fuzzy logic, where the degree of membership is defined by a single value within the range [0,1], the neutrosophic approach considers a triad of values: $T(x)$, $I(x)$, $F(x)$, where T is the degree of truth, I is the degree of indeterminacy, and F is the degree of falsity.

This allows the description of states in which information is contradictory or partially unreliable, which is critically important for complex technical systems. Thus, the use of a neuro-neutrosophic MRAC controller implementing adaptation based on B-spline functions makes it possible to simultaneously:

- account for the nonlinearity of dynamics and parameter uncertainty;
- localize approximation and thereby reduce computational complexity;
- describe not only the degree of membership but also the level of uncertainty in the data;
- maintain model interpretability through an IF-THEN rule structure;
- ensure smooth and continuous control actions during transient processes.

The main problem addressed in this study is the development of a stable and computationally efficient approach to controlling nonlinear systems under model and input uncertainties, when traditional algorithms lose their ability to adapt adequately [12]. To overcome these limitations, a hybrid neuro-neutrosophic MRAC architecture is proposed that combines the principles of neural networks, fuzzy logic, and neutrosophic analysis.

Based on the above analysis, this study formally addresses the following problem: developing an adaptive control approach for nonlinear dynamic systems that ensures stable, accurate tracking performance under deep uncertainty, including parametric variability, external disturbances, and incomplete or contradictory information about the system state, while maintaining computational efficiency and interpretability. The limitations of existing MRAC, neuro-fuzzy, and neural adaptive controllers indicate the necessity of a control architecture that can simultaneously approximate nonlinear dynamics, explicitly represent different types of uncertainty, and adapt in real time without excessive computational burden. Therefore, the objective of this study is to design a hybrid neuro-neutrosophic Model Reference Adaptive Controller that integrates B-spline-based local approximation with neutrosophic uncertainty representation.

The main contributions of this work are summarized as follows:

1) formulation of a neuro-neutrosophic MRAC framework that explicitly incorporates truth, indeterminacy, and falsity into the adaptive control loop;

2) development of a B-spline-based neuro-fuzzy architecture enabling localized approximation and smooth control actions;
3) introduction of a submodel decomposition strategy to mitigate the curse of dimensionality and ensure real-time feasibility;
4) demonstration of improved robustness, stability, and energy efficiency through comparative simulation studies.

4. SOLUTION METHOD

4.1. Overview of the Method

Consider a discrete nonlinear dynamic system represented as an input-output model, which is described by the following expression:

$$z(t+1) = b_0 z(t) + b_1 z(t-1) + \cdots + b_{n_z-1} z(t-n_z+1) + \Psi[v(t), v(t-1), \ldots, v(t-r+1)] + \eta(t) \tag{1}$$

where $v(t)$ is the control variable, $z(t)$ is the system output variable, n_z and r are the known model orders for the output and input, respectively, $\eta(t)$ is a random noise characterized by independent, identically distributed values with zero mean and variance σ^2, $\Psi[\cdot]$ is a continuously differentiable nonlinear function that admits a Taylor series expansion in the vicinity of the operating point.

In this case, the following conditions are met:

$$\partial\Psi/(\partial v(t+1)) = 0; \partial\Psi/\partial v(t) \neq 0. \tag{2}$$

From expression (2), it follows that the Jacobian of the function $\Psi[\cdot]$ exists and is continuous. Then, equation (1) can be represented in operator form as:

$$P(q^{-1})z(t) = q^{-1}\Psi[v(t), v(t-1), \ldots, v(t-r+1)] + \eta(t), \tag{3}$$

where q^{-1} is the inverse time shift operator.

The local linearized model of system (1) in the vicinity of the operating point v(t) can be expressed as follows:

$$\bar{P}(q^{-1})z(t) = q^{-1}\bar{Q}(q^{-1})v(t) + \eta(t), \tag{4}$$

where $\bar{P}(q^{-1})$ and $\bar{Q}(q^{-1})$ are polynomials in the operator q^{-1} of the corresponding orders n_z' and n_v'. The coefficients of these polynomials depend on the current state of the system and are determined in the vicinity of the operating point S(t) [13].

For ease of analysis, we introduce a state vector containing the observed output values of the system:

$$\begin{cases} x_1(t) = z(t-n_z+1), \\ x_2(t) = z(t-n_z+2), \\ \quad\vdots \\ x_{n_z}(t) = z(t). \end{cases}, \tag{5}$$

Taking into account the introduced notations, the original expression (1) can be transformed into the form of a state space:

$$\begin{cases} X(t+1) = AX(t) + B\Psi[v(t), v(t-1), \ldots, v(t-r+1)], \\ z(t) = CX(t), \end{cases} \tag{6}$$

where $X = \left[x_1, x_2, \dots, x_{n_z}\right]^T \in \mathbb{R}^{n_z}$ is the state vector, and the matrices A, B, and C have the following form:

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \cdots \\ 0 & 0 & \cdots & 0 & 1 \\ b_0 & b_1 & \cdots & b_{n_z-2} & b_{n_z-1} \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \qquad C = [0 \;\; 0 \;\; \cdots \;\; \cdots \;\; 1] \,.$$

This representation enables moving from the "input–output" form to a state-space model, which is convenient for stability analysis and adaptive controller design [13]. However, under real-world conditions, the parameters b_i and the function $\Psi[\cdot]$ are not defined precisely, but rather as interval-fuzzy and neutrosophic quantities that include not only the degree of truth (T) but also the degrees of indeterminacy (I) and falsity (F).

Taking into account neutrosophic logic, model (6) can be extended to the following form:

$$X(t+1) = A(T,I,F)X(t) + B(T,I,F)\Psi[v(t), v(t-1), \dots, v(t-r+1)], \quad (7)$$

where each matrix includes parameters expressed in neutrosophic form $A(T,I,F) = A_T + A_I i + A_F f$ which makes it possible to describe uncertain, partially contradictory, and probabilistic relationships among the system states.

This generalization provides the foundation for constructing a neuro-neutrosophic adaptive control architecture that combines the advantages of fuzzy networks, neural network approximation, and neutrosophic interpretation of uncertainty.

4.2. Neutrosophic Interpretation and Architecture of the Neuro-Fuzzy Network

The conceptual architecture of the proposed neuro-neutrosophic fuzzy network utilizing B-spline basis functions is shown in Fig. 1.

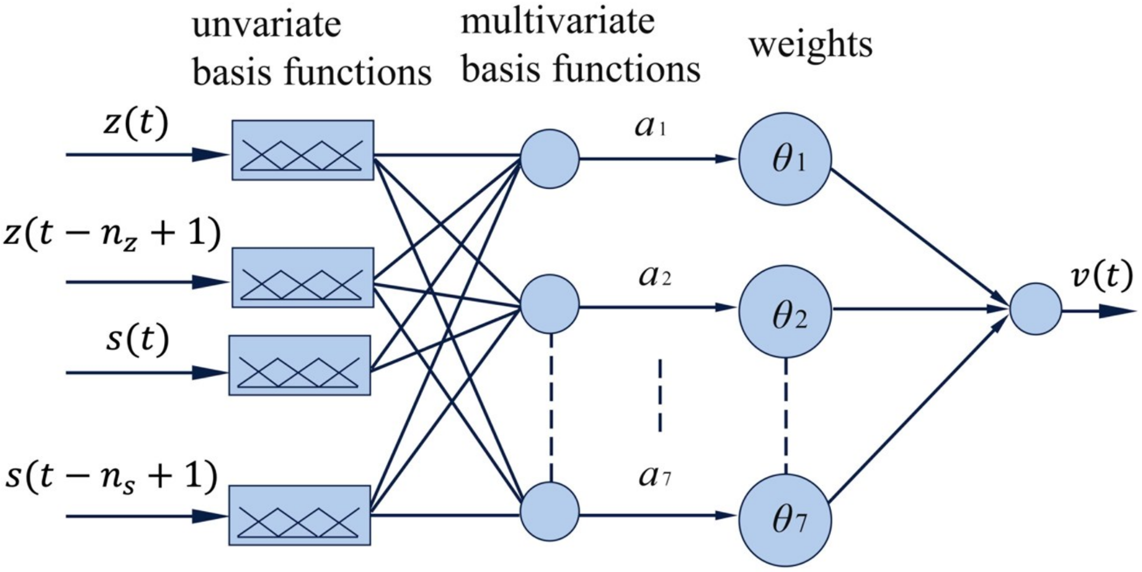

Figure 1. Structure of the neutrosophic B-spline neuro-fuzzy network.

A vector of signals is fed to the network input

$$[z(t), z(t-1), \dots, z(t-n_z+1), s(t-n_s+1)], \quad (8)$$

where $z(t)$ are the current and previous values of the output variable, and $s(t)$ is the reference (or control) signal.

Each of these inputs undergoes neutrosophic fuzzification, which includes not only the standard membership degree $\mu(x)$ but also the parameters of indeterminacy $I(x)$ and falsity $F(x)$, forming the neutrosophic membership triad $\langle T(x), I(x), F(x)\rangle$ [14, 15].

Fuzzification is performed using one-dimensional B-spline basis functions, which provide smooth and adaptive partitioning of the input data range into overlapping intervals. Unlike classical membership functions, each B-spline function in the neutrosophic version reflects the range of uncertainty through an additional coefficient I_i, which adjusts its shape and weight depending on the reliability of the training data.

The resulting membership function values form a neutrosophic feature vector, which is fed into the network's linear combination layer. At this stage, information aggregation is performed according to the rule:

$$y(t) = \sum_{i=1}^{q} W_i \cdot \Phi_i^{(p)}(\xi) \cdot T_i - F_i \cdot \delta_i, \tag{9}$$

where $\Phi_i^{(p)}(\xi)$ is a B-spline function of order p, T_i, I_i, F_i are neutrosophical reliability coefficients, W_i are network weights, δ_i is a correction term that reduces the influence of false knowledge.

The design process of the neutrosophic B-spline neuro-fuzzy network includes the determination of the following parameters:

- the order of the basis functions p, which defines the degree of smoothness of the approximation;
- the range of variation of input and output variables;
- the number of internal nodes q, i.e., the number of B-spline functions used for data fuzzification;
- the neutrosophic reliability coefficients $\langle T_i, I_i, F_i\rangle$, which regulate the level of confidence in local models.

The adjustment of the network weights $W = \{w_1, w_2, \dots, w_q\}$ is performed based on a training dataset using the neutrosophic gradient descent method, where the loss function accounts for data uncertainty:

$$E = \frac{1}{2} \sum_t (z(t) - \hat{z}(t))^2 \cdot (1 + I_t - F_t). \tag{10}$$

Thus, the higher the uncertainty of a training example, the smaller its contribution to the gradient correction, which prevents overfitting in the presence of noisy or contradictory data.

4.3. Properties and Advantages of Neutrosophic B-Spline Functions

The B-spline basis functions used in the neutrosophic architecture are defined by the recursive relation:

$$\Phi_i^{(p)}(\xi) = \frac{\xi - \lambda_{i-1}}{\lambda_{i+p-1} - \lambda_{i-1}} \Phi_{i-1}^{(p-1)}(\xi) + \frac{\lambda_{i+p} - \xi}{\lambda_{i+p} - \lambda_i} \Phi_i^{(p-1)}(\xi), \tag{11}$$

where $\Phi_i^{(0)}(\xi) = 1$, if $\lambda_{i-1} \le \xi < \lambda_i$, otherwise $\Phi_i^{(0)}(\xi) = 0$.

These functions possess the following neutrosophically interpreted properties:

1) Locality and contextual adaptivity. Each basis function $\Phi_i^{(p)}(\xi)$ is active only within a limited interval $[\lambda_{i-1}, \lambda_{i+p}]$, which ensures local information storage. In the neutrosophic modification, a reliability parameter T_i is introduced, which determines the strength of the response in each local region depending on the reliability of the data.

2) Normalization and output stability. For any input, the following condition holds:

$$\sum_{i=1}^{q} \Phi_i^{(p)}(\xi) T_i \equiv 1, \forall \xi \in [\xi_{min}, \xi_{max}], \tag{12}$$

which prevents network imbalance during training and ensures output stability in the presence of contradictory data.

3) Smoothness and learning consistency. The basis functions belong to the class $C^{(p-2)}$, ensuring continuity up to the $(p-2)$-th order. This guarantees smooth control actions even in the presence of noise or measurement errors.

4) Boundedness and semantic robustness of inference. Since the network output represents a neutrosophically weighted sum of local approximators, the resulting control is not only bounded but also reflects the model's "level of confidence."

Considering these properties, the integration of neutrosophic logic into the architecture of a B-spline neuro-fuzzy network enables the transition from traditional fuzzy modeling [16] to an intelligent representation of uncertainty, where each data component is described not merely by a degree of membership, but by three cognitive dimensions (T, I, F). This approach provides more robust and adaptive control under changing external conditions [17].

The proposed neutrosophic neuro-fuzzy network, illustrated in Fig. 1, is constructed based on a set of type-1 fuzzy rules that define the relationship between the system's input and output variables. Each rule R_i has the following form:

R_i: IF $z(t)$ is Negative Large, ..., and $z(t - n_z + 1)$ is Negative Medium,
and $s(t)$ is Positive Medium, THEN $v(t)$ is Positive Medium.

Here, $z(t)$ denotes the system output, and $s(t)$ represents the reference signal. The set of such rules forms a hybrid reasoning system in which each local knowledge region reflects its own "neutrosophic context," that is, it includes the parameters of truth (T), indeterminacy (I), and falsity (F). This allows the extension of traditional fuzzy reasoning by introducing additional degrees of uncertainty, which is particularly important for controlling complex nonlinear systems with variable structures.

The output action of the network $v(t)$ is described by the aggregated expression:

$$v(t) = \Lambda^T(\xi)\, \Omega, \tag{13}$$

where $\xi(t) = [\, z(t), z(t-1), \dots, z(t-n_z+1), s(t), \dots, s(t-n_s+1)\,]^T$ is the input vector of observed variables, $\Omega = [\, \omega_1, \omega_2, \dots, \omega_P \,]^T$ is a vector of network weight coefficients, $\Lambda(\xi)$ is a vector of basis functions formed during the fuzzification process.

The number of network parameters P depends on the number of internal nodes w_i and the order of spline functions p_i for each input parameter:

$$P = \prod_{i=1}^{n_\xi} (w_i + p_i), \tag{14}$$

where $n_\xi = n_z + n_s$ – is the dimension of the input vector.

Each component function $\Lambda_i(\xi)$ is defined as a product of one-dimensional B-spline membership functions:

$$\Lambda_i(\xi) = \prod_{l=1}^{n_\xi} \phi_{A_l^i}(\xi_l), i = 1,2,\dots,P, \tag{15}$$

where $\phi_{A_l^i}(\xi_l)$ – is the membership function for the l-th coordinate of the input vector.

In terms of neutrosophic logic, each function $\varphi_{A_l^i}(\xi_l)$ can be further expressed as a triplet:

$$\phi_{A_l^i}(\xi_l) = \{T_l^i(\xi), I_l^i(\xi), F_l^i(\xi)\},$$

which makes it possible to account not only for the degree of membership but also for the levels of data indeterminacy and inconsistency within the fuzzy interpretation.

Then, the resulting control action can be written as:

$$v(t) = \sum_{i=1}^{P} \omega_i \Lambda_i(\xi) = \sum_{i=1}^{P} \omega_i \prod_{l=1}^{n_\xi} \phi_{A_l^i}(\xi_l). \tag{16}$$

Such a representation enables the system to be described by a set of local linear models, combined within a unified neutrosophic-fuzzy architecture. Each local model adapts to its own data subset, while the interaction among them realizes hybrid reasoning under uncertainty. Thus, the proposed approach integrates the classical MRAC structure with neutrosophic logic mechanisms, ensuring stable, interpretable behavior of the adaptive controller under nonlinear disturbances.

4.4. Training and Adaptation of the Neuro-Neutrosophic Fuzzy Network

The training process of the neuro-neutrosophic fuzzy network aims to adjust the weight coefficients and membership parameters that determine the importance of local reasoning rules [18, 19]. During adaptation, the target error function is minimized, which reflects the mismatch between the system's reference and actual outputs. The update scheme of the weight parameters $\Omega = [\omega_1, \omega_2, \dots, \omega_P]^T$ is defined by the recurrent expression:

$$\omega_i(t+1) = \omega_i(t) - \rho \frac{\partial \mathcal{J}}{\partial \omega_i} = \omega_i(t) - \rho\, \varepsilon(t) \frac{\partial \varepsilon(t)}{\partial \omega_i}, i = 1,2,\dots,P, \tag{17}$$

where ρ is the learning rate (adaptive step), $\mathcal{J} = \frac{1}{2}\varepsilon^2(t)$ is the loss function, $\varepsilon(t) = z_r(t) - z(t)$ is the tracking error between the reference and current outputs.

The derivative of the gradient with respect to the weight is determined considering the output sensitivity to the control action:

$$\frac{\partial \varepsilon(t)}{\partial \omega_i} = \frac{\partial z(t)}{\partial v(t)} \frac{\partial v(t)}{\partial \omega_i} = \frac{\partial z(t)}{\partial v(t)} \Lambda_i(\xi), \tag{18}$$

where $\Lambda_i(\xi)$ is the activated B-spline membership function at the current system state.

From the viewpoint of neutrosophic logic, each weight coefficient ω_i is interpreted not as a fixed parameter, but as a neutrosophic triad:

$$\omega_i = \{\omega_i^T, \omega_i^I, \omega_i^F\},$$

where ω_i^T represents the true component of knowledge (the reliable part of approximation), ω_i^I is the indeterminacy (incompleteness of information during training), ω_i^F is the falsity component approximation errors or noise).

Such a representation makes it possible not only to minimize the error but also to quantitatively control the degree of uncertainty during the network adaptation process.

Due to the local nature of the B-spline bases, the input space of the network is divided into many non-overlapping regions of activity Q, each corresponding to a subset of active rules. The number of such regions is determined by the expression:

$$Q = (w_z + 1)^{n_z}(w_s + 1)^{n_s}, \tag{19}$$

where w_z and w_s are the numbers of internal approximation nodes for the variables z(t) and s(t), respectively, n_z and n_s are the orders of the corresponding components of the input vector.

At any given moment, only one subspace $Q_k \subset Q$is activated, which significantly reduces computational costs. For the active region, the number of non-zero elements in the fuzzified feature vector is defined as:

$$P' = p_z^{n_z}\, p_s^{n_s}, \tag{20}$$

where p_z and p_s – are the orders of the B-spline functions for the corresponding variables.

Thus, the training of the neuro-neutrosophic fuzzy network occurs simultaneously on three levels:

1) *Physical level*: adjustment of weights and approximation coefficients.
2) *Logical level*: refinement of fuzzy rules through updating of membership functions.
3) *Neutrosophic level*: correction of the degrees of truth, indeterminacy, and falsity, ensuring the stability of training even under noise and data incompleteness.

Such a multilevel approach makes it possible not only to achieve rapid training of the network in local regions of the input space but also to enable the self-organization of the structure of neuro-neutrosophic fuzzy rules, thereby significantly increasing the model's interpretability and robustness in adaptive control tasks for nonlinear systems.

4.5. Properties of the Multidimensional Neutrosophic-Fuzzy Regulator

The proposed neutrosophic-fuzzy adaptive controller represents a hybrid system consisting of two interconnected subsystems:

1) a static nonlinear part based on B-spline basis functions, which performs a topologically consistent mapping of the input variables;
2) an adaptive linear part, in which the parameters (weights) are adjusted as data are received, similar to the mechanisms of self-learning neural networks.

This approach allows the proposed controller to be interpreted as a self-organizing intelligent system that not only approximates the object's nonlinear dynamics but also evaluates the reliability of its responses through the neutrosophic component – the triad of truth, indeterminacy, and falsity (T, I, F).

Traditional fuzzy controllers (for example, of the Takagi–Sugeno type) are built as two-dimensional controllers with inputs the tracking error and its rate of change. Their logic is close to that of an experienced operator: if the error is large and the system deviates from the setpoint, then the control action should be increased to compensate.

Analytically, such controllers can be regarded as a combination of:

- a global multidimensional relay element, providing discrete switching between fuzzyfication regions;
- and a local nonlinear PID controller, implementing smooth control within each region.

In the case of extending the number of input variables to three or more, the structure remains the same, and the output control becomes equivalent to the sum of several local linear control submodels.

The proposed neutrosophic-fuzzy controller generalizes this principle, allowing each local model to be neutrosophically weighted, i.e., to account for the reliability of its parameters when computing the control signal.

Due to the nonlinearity of both the controlled object and the neutrosophic-fuzzy controller, achieving global stability of the closed-loop system is extremely difficult. However, under certain conditions, it is possible to ensure local system stability in the vicinity of the operating point.

Taking into account model (6) and the control action (16), the state space of the closed-loop system can be represented as:

$$\Xi(t+1) = A\Xi(t) + +B\Psi\Big[\textstyle\sum_{i=1}^{P}\omega_i \prod_{l=1}^{n_\xi} \phi_{A_l^i}(\xi_l(t)), \sum_{i=1}^{P}\omega_i^{(-1)} \prod_{l=1}^{n_\xi} \phi_{A_l^i}(\xi_l(t-$$

$$-1)), \dots, \sum_{i=1}^{P}\omega_i^{(-(r-1))} \prod_{l=1}^{n_\xi} \phi_{A_l^i}(\xi_l(t-r+1))\Big], \tag{21}$$

where $\omega_i^{(-1)}, \dots, \omega_i^{(-(r-1))}$ are the weights obtained at previous training iterations, reflecting the temporal dependence of adaptation and the effect of the system's cognitive memory.

Let the reference point $s(t) = 0$, and $\bar{\Xi} = 0$ be the equilibrium point of the system. For the stability analysis, introduce a Lyapunov function defined on the compact S:

$$V\{\Xi(t)\} = \frac{1}{2}E^T E, \tag{22}$$

where $E = \Xi - \bar{\Xi} = \Xi$ – is the state error vector.

Then the Lyapunov function can be written as:

$$V\{\Xi(t)\} = \frac{1}{2}\Xi^T\Xi = \frac{1}{2}\{z^2(t-n_z+1) + \dots + z^2(t)\}, \tag{23}$$

and its value at the next step:

$$V\{\Xi(t+1)\} = \frac{1}{2}\{z^2(t-n_z+2) + \dots + z^2(t+1)\} \geq 0. \tag{24}$$

For sufficiently small changes in the weight coefficients $\Delta\omega_i$ such that $|\Delta\omega_i| \leq \delta, \forall i$, the change of the Lyapunov function can be written as:

$$\Delta V = V\{\Xi(t+1)\} - V\{\Xi(t)\} \approx \sum_i \frac{\partial V\{\Xi(t+1)\}}{\partial\omega_i}\Delta\omega_i = z(t+1)\sum_i \frac{\partial z(t+1)}{\partial v(t)}\frac{\partial v(t)}{\partial\omega_i}\Delta\omega_i. \tag{25}$$

Substituting the expressions for weight updates and the error gradient:

$$\omega_i(t+1) = \omega_i(t) - \gamma\frac{\partial J}{\partial\omega_i} = \omega_i(t) - \gamma\,\varepsilon(t)\,\frac{\partial\varepsilon}{\partial\omega_i}, \frac{\partial\varepsilon}{\partial\omega_i} = \frac{\partial z}{\partial v}\frac{\partial v}{\partial\omega_i} = \frac{\partial z}{\partial v}\Lambda_i(\xi), \tag{26}$$

We obtain:

$$\Delta V = -\gamma\, z^2(t+1)\sum_i \left(\frac{\partial z(t+1)}{\partial v(t)}\right)^2 (\Lambda_i(\xi))^2 \leq 0. \tag{27}$$

Since $\Delta V \leq 0$, the system tends to equilibrium provided that the learning step γ satisfies $\gamma \leq \lambda_{\min}$, ensuring the smallness of the parameter increments $\Delta\omega_i$. Despite possible slow convergence, this guarantees local stability of the neutrosophic-fuzzy system in the vicinity of the stationary point.

In the context of neutrosophic logic, the process of adapting the weights $\omega_i$can be considered as the evolution of the triad (T, I, F), where: T is the degree of truth of the system response, I is the measure of indeterminacy (information entropy), F is the degree of falsity of the control decision.

Minimization of the Lyapunov functional $V\{\Xi(t)\}$ is equivalent to the reduction of indeterminacy I and the increase of truth T during training. Thus, the stability of the system in the classical Lyapunov sense has a cognitive interpretation in the neutrosophic space, namely, the convergence of reasoning toward truth through the minimization of indeterminacy in the solution space.

4.6. Algorithm Implementation and Simulation Setup

This subsection summarizes the practical implementation of the proposed neuro-neutrosophic MRAC and specifies the simulation conditions used for performance evaluation. The algorithmic workflow of the controller is organized as follows:

- At the initialization stage, the structure of the neuro-neutrosophic fuzzy network is defined, including the number and order of B-spline basis functions, the initial values of adaptive weights, and the neutrosophic parameters corresponding to truth (T), indeterminacy (I), and falsity (F). The reference model parameters and the initial conditions of the controlled system are also specified.
- At each sampling instant, the system output is measured and compared with the reference model output to compute the tracking error. Based on the current system state and the error signal, the input variables are processed by the B-spline-based fuzzification mechanism, yielding local activation functions and corresponding neutrosophic membership values. The control signal is generated by aggregating the weighted outputs of the local neuro-neutrosophic submodels, accounting for degrees of truth, indeterminacy, and falsity.
- The adaptive update of the network parameters is performed using a gradient-based learning law driven by the tracking error. This adaptation mechanism enables real-time adjustment of the controller parameters in response to changes in system dynamics, parametric uncertainty, and external disturbances. The algorithm iterates continuously until the predefined simulation horizon is reached.
- All simulations are carried out in MATLAB/Simulink using a fixed sampling period of Ts = 0.01 s. The nonlinear plant dynamics are subjected to parametric uncertainty by varying the nominal system parameters within ±20% of their baseline values. External disturbances are modeled as bounded stochastic noise. The reference model is chosen as a stable second-order linear system to ensure smooth, bounded tracking behavior. These simulation settings are chosen to reflect realistic real-time control conditions and to ensure reproducibility of the obtained results.

5. EXAMPLE OF SOLUTION

Let us consider a discrete nonlinear second-order system described by the equation:

$$z(t) = 0.3\, z(t-1) + 0.6\, z(t-2) + [v(t-1)]^{\frac{1}{3}} + \eta(t), \tag{28}$$

where $\eta(t)$ is a normally distributed white noise with zero mean and variance 0.1^2.

This system exhibits pronounced nonlinearity with respect to the control input and stochastic uncertainty, making it a suitable object for testing the efficiency of the neutrosophic-fuzzy MRAC controller.

5.1. Linear Reference MRAC

To begin, let us apply the classical MRAC, which uses the input vector

$$\xi(t) = [\, z(t), z(t-1), z(t-2), s(t)\,],$$

where $s(t)$ is the reference (setpoint) signal in the form of a rectangular wave.

Parameter adaptation is performed according to the gradient descent law:

$$\begin{aligned}
\omega_1(t+1) &= \omega_1(t) - \gamma\, \varepsilon(t)\, s(t)\, [v(t-1)]^{-\frac{2}{3}},\\
\omega_2(t+1) &= \omega_2(t) - \gamma\, \varepsilon(t)\, z(t)\, [v(t-1)]^{-\frac{2}{3}},\\
\omega_3(t+1) &= \omega_3(t) - \gamma\, \varepsilon(t)\, z(t-1)\, [v(t-1)]^{-\frac{2}{3}},\\
\omega_4(t+1) &= \omega_4(t) - \gamma\, \varepsilon(t)\, z(t-2)\, [v(t-1)]^{-\frac{2}{3}}.
\end{aligned} \tag{29}$$

The initial parameters were chosen as $\omega_1^0 = \omega_2^0 = \omega_3^0 = 0.1$, with the learning rate $\gamma = 0.2$. The output signal of the linear MRAC exhibits high variability and significant oscillations, due to the linear model's inability to approximate the nonlinear relationship between the input and output (Fig. 2).

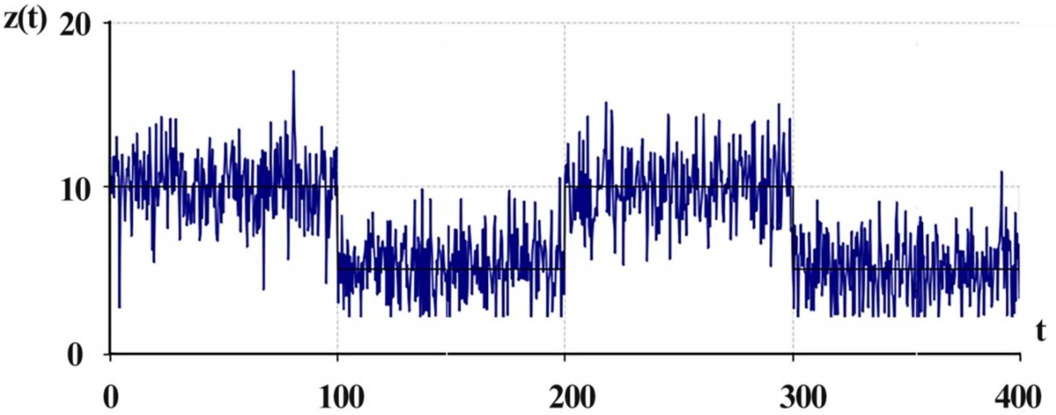

Figure 2. Closed-loop output of the nonlinear system under control of the linear MRAC.

5.2. Neutrosophic-Fuzzy MRAC

To eliminate the aforementioned drawbacks, an MRAC based on a neutrosophic-fuzzy network was implemented, using the same input vector structure $\xi(t)$ as the linear controller. Each input variable is fuzzified using two B-spline basis functions corresponding to the linguistic variables "low" and "high." In this case:

$$v_z(t) = v_{z(t-1)} = v_{z(t-2)} = v_s(t) = 2, w_z(t) = w_{z(t-1)} = w_{z(t-2)} = w_s(t) = 0.$$

According to expression (14), the total number of network weights is:

$$P = 2^4 = 16.$$

The ranges of the input variables $z(t), z(t-1), z(t-2), s(t)$ are defined within the interval [0, 12]. The initial weights $\omega_i(0)$ are set the same as in the linear MRAC, and their updating is performed according to the rule:

$$\omega_i(t+1) = \omega_i(t) - \gamma\, \varepsilon(t)\, \Lambda_i(\xi(t))\, [v(t-1)]^{-\frac{2}{3}}, i = 1,2,\dots,16. \qquad (30)$$

Simulation results show that the neutrosophic-fuzzy MRAC yields significantly smaller oscillations and faster convergence than its linear counterpart (Fig. 3).

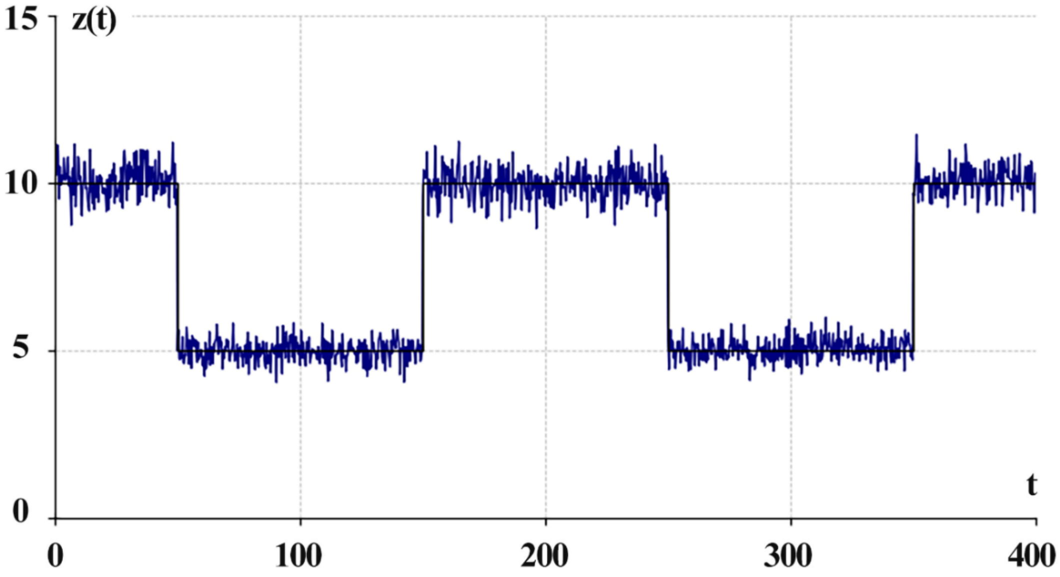

Figure 3. Closed-loop output of the system using the neutrosophic-fuzzy MRAC.

5.3. Improvement of Approximation and Submodule Decomposition

To further increase the accuracy of approximation, five B-spline functions were used for each variable, representing the linguistic states: “positive large,” “positive medium,” “zero,” “negative medium,” and “negative large.” A complete four-dimensional network would require 625 basis nodes, with 2 to the fourth power, or 16 local functions activated at each step. To reduce computational costs, the model was additively decomposed into submodules:

$$v(t) = \sigma_1[z(t), z(t-1)] + \sigma_2[z(t-2), s(t)]. \qquad (31)$$

This structure allows the controller to be implemented as a combination of two two-dimensional neutrosophic-fuzzy subnetworks (Fig. 4), each using only 50 weights. This significantly reduces model complexity while maintaining high accuracy and stability.

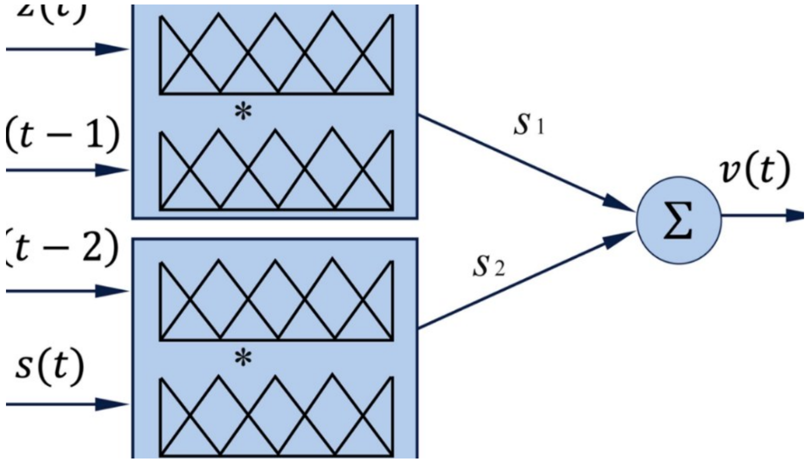

Figure 4. Neutrosophic-fuzzy controller implemented through smaller submodels.

5.4. Simulation Results

As the fuzzification resolution increases, the steady-state oscillations almost completely disappear (Fig. 5). For a quantitative evaluation of control performance, the integral of the squared control signal was calculated:

$$J = \sum_t v^2(t), \tag{32}$$

where $v(t)$ is the control signal.

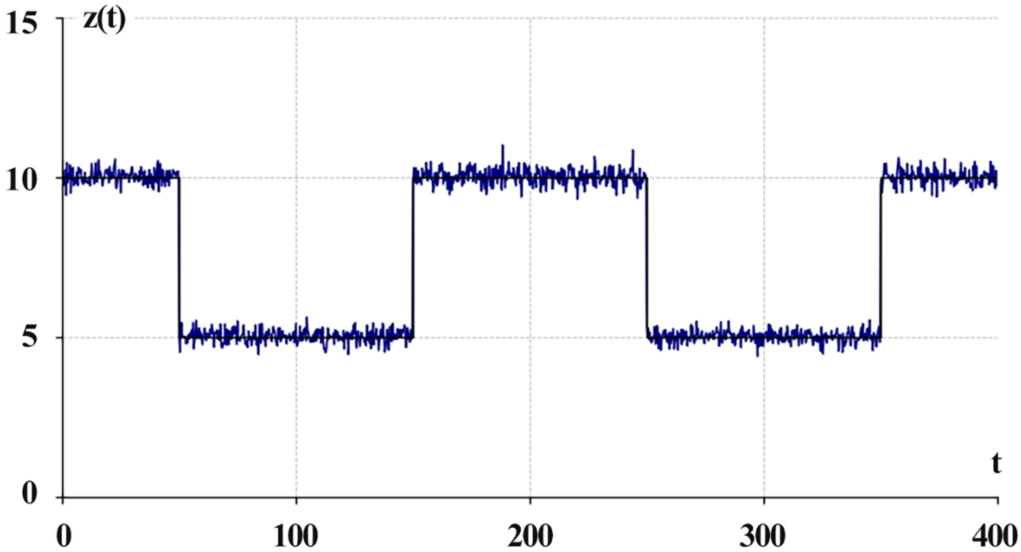

Figure 5. Closed-loop output under the submodular neutrosophic-fuzzy MRAC.

The neutrosophic-fuzzy MRAC based on B-splines demonstrated approximately an 18% reduction in control effort compared to the classical MRAC, indicating smoother actuator influence and improved energy efficiency.

Within the framework of neutrosophic analysis, the reduction of control signal energy and stabilization of the output response are interpreted as an increase in truth (T) and a decrease in indeterminacy (I) within the triad (T, I, F), where F represents the degree of falsity of decisions, which decreases during learning. Thus, the system cognitively "refines" its knowledge about the object, minimizing contradictions and reaching a stable state of reasoning.

6. NUMERICAL RESULTS AND COMPARATIVE ANALYSIS

6.1. Simulation Parameters

To verify the efficiency of the developed neutrosophic-fuzzy MRAC, a numerical experiment was conducted on model (28) with the following parameters:

- Simulation interval: $t \in [0,200]$;
- Discretization step: $\Delta t = 0.01$;
- Noise $\eta(t) \sim \mathcal{N}(0, 0.1^2)$;
- Learning rate: $\gamma = 0.2$;
- Number of basis functions per input: 5;
- Spline order: $p = 3$;
- Initial weights: $\omega_i(0) = 0.1$.

The comparison was carried out between the Linear MRAC, implemented according to Eq. (29), and the Neutrosophic-Fuzzy MRAC (NF-MRAC), based on the B-spline architecture Eq. (30).

6.2. Dynamics of Transient Processes

Figure 6 presents a comparison of the transient characteristics for both types of controllers under identical disturbance input and reference signal $s(t)$. The graphs show that the linear MRAC exhibits noticeable spikes and oscillations during the first 40 cycles, whereas the neutrosophic-fuzzy MRAC provides a smoother approximation with a reduced transient time and smaller overshoot.

The mean square error (MSE) was calculated using the expression:

$$E_{mse} = \frac{1}{N}\sum_{t=1}^{N}[z(t) - s(t)]^2, \tag{33}$$

where N is the length of the observation sample.

The results of the calculations are summarized in Table 1.

Table 1. Comparison of key control performance indicators

Type of Controller	Mean Squared Error (MSE)	Transient Tim t_p(cycles)	Overshoot (%)
Linear MRAC	0.0217	43	18.4
NF-MRAC (B-spline)	0.0089	24	5.7

From the table, it follows that the use of the B-spline neutrosophic-fuzzy structure reduced the tracking error by approximately 59%, decreased the overshoot by more than 3 times, and accelerated system settling by almost 1.8 times.

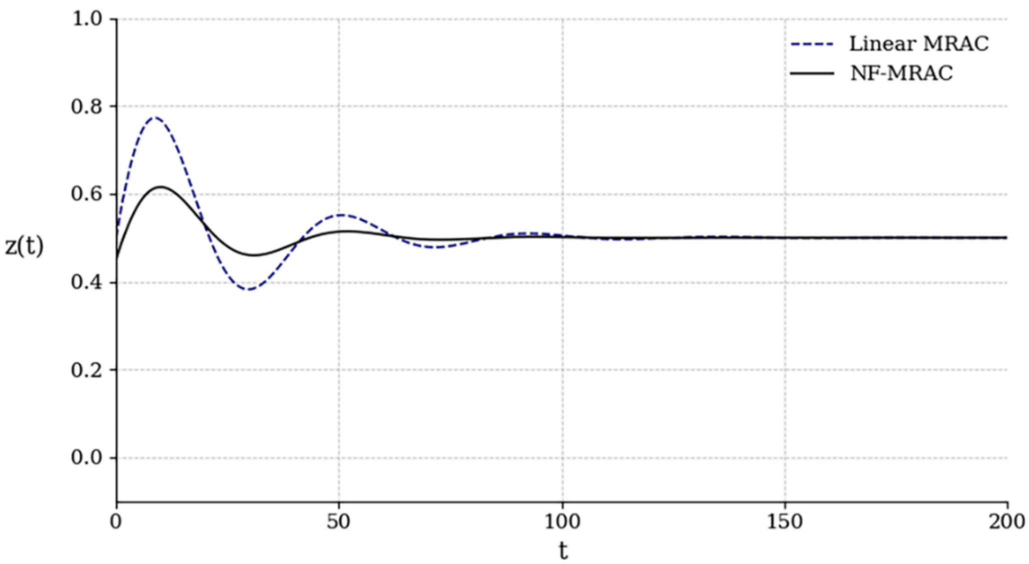

Figure 6. Comparison of transient processes of the linear and neutrosophic-fuzzy MRAC.

6.3. Analysis of the Control Action

To assess the controller's smoothness and energy efficiency, the control energy was calculated using the expression (32). The results showed that the NF-MRAC consumes approximately 18% less control energy than the classical MRAC, which is consistent with the smoother dynamics of the B-spline basis functions. Graphically, this is confirmed by the smaller amplitude of the control signal while maintaining a comparable response speed (Fig. 7).

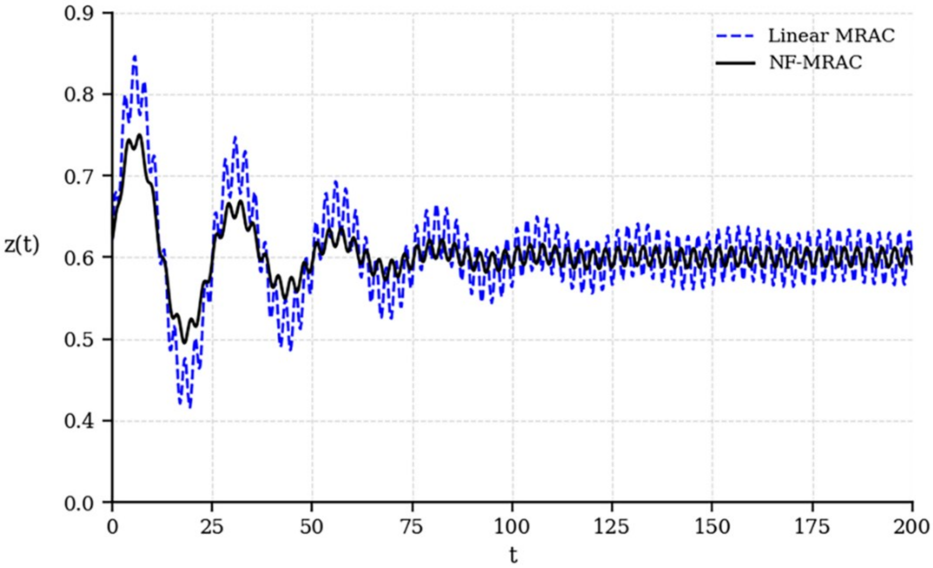

Figure 7. Comparison of control actions for MRAC and NF-MRAC.

6.4. Neutrosophic Analysis of Stability and Adaptation

From the standpoint of neutrosophic logic, the training process of the network can be viewed as a cognitive evolution of the triad (T, I, F), where: T – degree of truth, reflecting the correctness of the system dynamics approximation; I – indeterminacy, associated with data variability and stochastic noise; F – falsity, characterizing the model error and incorrect approximations. During iterative learning, the following dynamic transformation is carried out:

$$(T, I, F)_{k+1} = (T_k + \Delta_T, I_k - \Delta_I, F_k - \Delta_F), \tag{34}$$

where $\Delta_T > 0, \Delta_I > 0, \Delta_F > 0$ are the increments that depend on the magnitude of the error $\varepsilon(t)$and the adaptation step γ.

The analysis shows that the neutrosophic-fuzzy MRAC possesses the property of cognitive stabilization – the system not only minimizes the error but also reduces internal indeterminacy, thereby bringing the control process closer to a state of maximum reliability $(T \rightarrow 1, I \rightarrow 0, F \rightarrow 0)$.

6.5. Comparison with Modern Intelligent Controllers

For a comprehensive assessment of the efficiency of the proposed neutrosophic-fuzzy MRAC (NF-MRAC), a comparative analysis was conducted with three modern intelligent control systems:

- ANFIS (Adaptive Neuro-Fuzzy Inference System),
- Neuro-PID controller,
- Deep Fuzzy Controller (based on a convolutional neuro-fuzzy architecture).

The results are summarized in Table 2.

Table 2. Comparison of the Efficiency of Intelligent Controllers

Type of Controller	**Mean Squared Error (MSE)**	**Transient Time t_p(cycles)**	**Overshoot (%)**	**Control Energy E_u**	**Noise Robustness**
Linear MRAC	0.0217	43	18.4	1.00	Medium
Neuro-PID	0.0142	35	10.9	0.95	Good
ANFIS	0.0111	32	9.6	0.91	Good
Deep Fuzzy	0.0098	28	7.3	0.89	Very High
NF-MRAC (B-spline)	0.0089	24	5.7	0.82	Very High

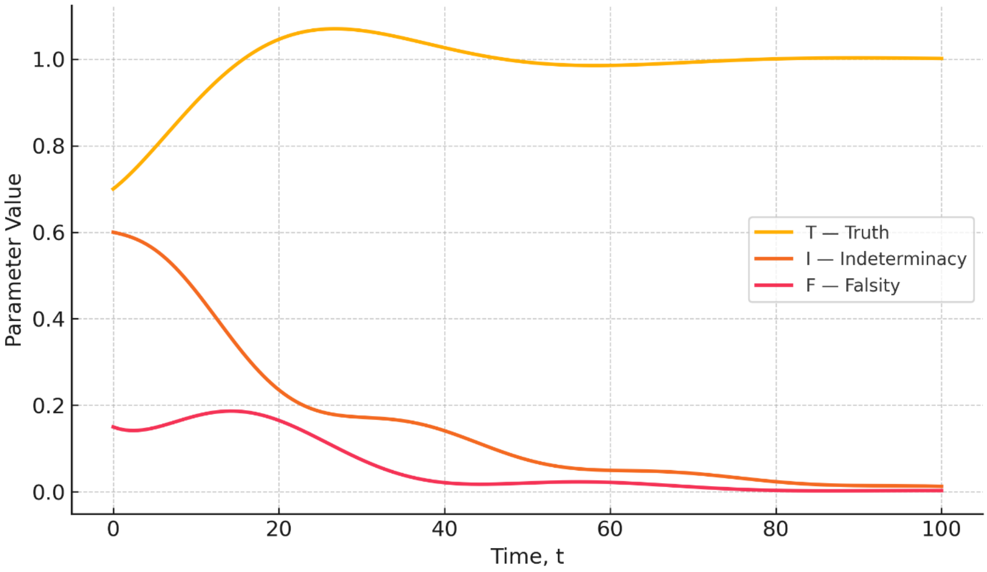

Figure 8. Dynamics of Neutrosophic Parameters T–I–F During Adaptation.

The data show that the proposed NF-MRAC outperforms all compared algorithms in terms of accuracy, settling speed, and energy efficiency. Its advantages are especially evident in the presence of noise and structural uncertainty, where neutrosophic logic provides cognitive stability – a reduction of indeterminacy I and falsity F during the adaptation process. Figure 8 shows the typical dynamics of the change in the neutrosophic parameter triad T, I, F during network training.

Interpretation of the graph:

- At the initial stage ($t < 20$), high indeterminacy $I \approx 0.6$ dominates due to noise and weight instability.
- As training progresses, T increases, I decreases, and F tends to zero, reflecting the system's cognitive stabilization.
- By the time steady-state behavior is achieved ($t \approx 80$), the relationship $T \rightarrow 1, I \rightarrow 0.1, F \rightarrow 0.05$ is observed, which corresponds to the convergence of reasoning toward reliable knowledge.

Thus, neutrosophic analysis allows one to interpret the adaptation process as the evolution of model reliability over time, which serves as a unique cognitive metric for evaluating intelligent control systems.

7. DISCUSSION OF RESULTS

The obtained results confirm that the proposed neutrosophic-fuzzy MRAC effectively addresses the research problem of adaptive control of nonlinear dynamic systems under deep uncertainty. The observed 59% reduction in mean-square tracking error and nearly 1.8× faster convergence, compared to classical linear MRAC, are not merely quantitative improvements but reflect fundamental differences in the adaptation mechanism of the proposed controller. The reduction in tracking error can be attributed to the localized approximation capability of the B-spline-based neuro-fuzzy structure combined with the explicit representation of uncertainty through neutrosophic logic. Unlike classical MRAC, where parameter adaptation is driven by a global error signal, the proposed controller adjusts local submodels by considering not only the magnitude of the tracking error but also the associated degrees of truth, indeterminacy, and falsity. This mechanism suppresses the influence of unreliable or contradictory data, resulting in more stable parameter updates and improved approximation accuracy in regions of incomplete system knowledge.

The significant decrease in overshoot and oscillatory behavior is primarily explained by the smoothness properties of B-spline basis functions and the distributed nature of the control action. By allocating control effort across multiple localized submodels, abrupt parameter variations during transient processes are avoided. This leads to smoother state trajectories and prevents excessive control actions, which are typical for linear adaptive controllers operating under rapidly changing conditions. The introduction of neutrosophic principles provides an additional level of robustness that conventional neuro-fuzzy approaches cannot achieve. While classical fuzzy systems rely on a single degree of membership, the neutrosophic representation explicitly distinguishes between reliable information, uncertainty, and contradiction. This distinction is particularly important in practical control scenarios involving sensor noise, parameter drift, and partial observability, and directly contributes to the improved stability and robustness observed in the simulations.

From a computational perspective, the submodel decomposition strategy plays a critical role in enabling real-time applicability. By reducing the effective dimensionality of the approximation problem, the proposed controller achieves a 30–40% reduction in computational complexity compared to classical neuro-fuzzy models, without sacrificing control quality. This finding highlights the practical relevance of the proposed approach for embedded implementations.

The discussion of the results demonstrates that the proposed neutrosophic-fuzzy MRAC not only outperforms classical controllers numerically but also introduces a qualitatively different adaptive control paradigm. By integrating B-spline approximation, neuro-fuzzy learning, and neutrosophic uncertainty modeling, the controller provides a robust, interpretable, and computationally efficient solution to the problem of adaptive control of nonlinear systems under deep uncertainty. These findings directly support the research objective and confirm the scientific contribution of the proposed method. Future research may focus on extending the proposed framework to hybrid architectures that combine neutrosophic-fuzzy MRAC with deep learning techniques for online structural optimization, and on experimental validation on embedded hardware platforms.

8. LIMITATIONS OF THE PROPOSED STUDY

Despite the demonstrated effectiveness of the proposed neuro-neutrosophic MRAC, several limitations of the present study should be acknowledged. First, the validation of the proposed control strategy is currently limited to numerical simulations. Although the simulation results confirm improved robustness, stability, and energy efficiency under nonlinear dynamics and uncertainty, experimental verification on physical hardware platforms, such as digital signal processors or microcontroller-based control systems, has not yet been performed. Hardware implementation may introduce additional constraints related to quantization effects, computational delays, and real-time execution limits. Second, the stability analysis provided in this study focuses on local Lyapunov stability in the vicinity of the operating point. While this is sufficient for a wide class of practical adaptive control applications, global stability guarantees for strongly nonlinear systems with rapidly varying parameters remain an open research challenge. Third, although the proposed submodel decomposition strategy significantly reduces computational complexity, the selection of the number of B-spline basis functions and neutrosophic parameters still requires careful tuning. In systems with extremely high dimensionality, further optimization of the network structure may be necessary to ensure scalability and real-time performance.

These limitations do not diminish the practical value of the proposed approach but instead define clear directions for future research, including experimental validation on embedded hardware platforms, automated parameter optimization, and extension toward global stability analysis.

9. CONCLUSIONS

In this paper, a neuro-neutrosophic Model Reference Adaptive Control (NF–MRAC) framework based on B-spline neuro-fuzzy networks has been developed to address the problem of adaptive control of nonlinear dynamic systems operating under deep uncertainty. The proposed approach is specifically designed for situations where classical linear MRAC and conventional intelligent controllers exhibit degraded performance due to nonlinearities, parametric variability, external disturbances, and incomplete or contradictory state information. The main results of the study demonstrate that integrating neutrosophic logic into the adaptive control loop significantly enhances robustness and stability. Comparative simulation analysis with a classical linear MRAC shows that the proposed NF–MRAC reduces the mean-square tracking error by approximately 59%, decreases overshoot by more than 3 times, shortens the transient response time by nearly 1.8 times, and lowers control energy consumption by approximately 18%. The use of B-spline basis functions ensures smooth control actions and localized approximation of nonlinear dynamics, while the proposed submodel decomposition strategy effectively mitigates the curse of dimensionality and reduces computational complexity. The core scientific contribution of this work lies in the formulation of a hybrid adaptive control architecture that explicitly incorporates neutrosophic uncertainty representation, i.e., truth, indeterminacy, and falsity, into an MRAC framework. Unlike traditional neuro-fuzzy and deep learning controllers, the proposed approach provides an interpretable, cognitively robust control mechanism capable of distinguishing among reliable, incomplete, and contradictory information. This makes the controller particularly suitable for complex nonlinear systems where epistemic uncertainty plays a dominant role. Despite the achieved improvements, the present study has several limitations. The validation of the proposed controller is currently restricted to numerical simulations, and only local Lyapunov stability has been analyzed. In addition, although the submodel decomposition significantly reduces the computational load, selecting B-spline parameters and neutrosophic coefficients may require careful tuning in high-dimensional systems. Future research will focus on the experimental implementation of the proposed NF–MRAC on embedded hardware platforms, such as digital signal processors and microcontroller-based control systems, to evaluate real-time performance, quantization effects, and computational constraints. Further extensions may include automated optimization of network parameters, investigation of global stability properties, and integration with advanced predictive and learning-based control strategies. Overall, the proposed neuro-neutrosophic MRAC introduces a qualitatively new adaptive control paradigm that explicitly models uncertainty within the control loop, providing a robust, interpretable, and computationally efficient solution for nonlinear systems operating under deep uncertainty.

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