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CONSTRUCTION AND ANALYSIS OF A NEW SEVEN-DIMENSIONAL CHAOTIC SYSTEM WITH COMPLEX DYNAMICAL BEHAVIORS

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ABSTRACT: This work presents a novel seven-dimensional hyperchaotic system with six Lyapunov exponents, three of which are positive, confirming chaotic behavior. The proposed system exhibits intricate dynamic behavior. This makes it highly suitable for various applications, particularly in secure communication. The system's key properties were analyzed using Lyapunov exponents, where the highest exponent is "6.68949", further confirming its strong chaotic nature. Additionally, bifurcation diagrams, phase portraits, equilibrium points, the Kaplan-Yorke dimension, and multistability were examined, providing a comprehensive understanding of its complex dynamics. Other important characteristics, including dissipativity, waveform analysis, sensitivity to initial conditions, and keyspace analysis, were also investigated to validate the system's robustness and applicability. A Mathematica model generated phase images of several strange attractors to support the theoretical findings.

ABSTRAK: Karya ini mempersembahkan sistem hiperchaotic tujuh dimensi novel dengan enam eksponen Lyapunov, tiga daripadanya positif, mengesahkan tingkah laku huru-hara. Sistem yang dicadangkan mempamerkan tingkah laku dinamik yang rumit. Ini menjadikannya sangat sesuai untuk pelbagai aplikasi, terutamanya dalam komunikasi selamat. Sifat utama sistem dianalisis menggunakan eksponen Lyapunov, di mana eksponen tertinggi ialah "6.68949", seterusnya mengesahkan sifat huru-hara yang kuat. Selain itu, gambar rajah bifurkasi, potret fasa, titik keseimbangan, dimensi Kaplan-Yorke, dan kestabilan berbilang telah diperiksa, memberikan pemahaman menyeluruh tentang dinamik kompleksnya. Ciri penting lain, termasuk dissipativity, analisis bentuk gelombang, kepekaan kepada keadaan awal dan analisis ruang kekunci, juga disiasat untuk mengesahkan keteguhan dan kebolehgunaan sistem. Menggunakan model MATHEMATICA, imej fasa beberapa penarik pelik dijana untuk menyokong penemuan teori.

KEYWORDS: *HyperChaotic , Seven Dimensional, Dynamic Analysis, Lyapunov exponent, Equilibrium points.*

1. INTRODUCTION

Chaos theory has found numerous practical applications in fields such as engineering, physics, medicine, and encryption [1–3]. Since Lorenz introduced the first 3-dimensional chaotic attractor in 1963 [4], researchers have extensively examined such systems due to their fascinating dynamics and potential uses. Over the decades, various 3D chaotic systems have been developed, including the Lü system, Chen system, Chua's circuit, and Liu system and the list goes on [5]. These systems have inspired the discovery of a wide range of chaotic attractors, such as scrolls, butterflies, and other intricate structures [6]. three-dimensional autonomous chaotic systems remain significant due to their simplicity and minimal computational requirements. However, with the growing demand for more reliable and secure applications, the focus has shifted toward higher-dimensional systems, which offer greater complexity and possibilities.

In 1976, Rössler introduced the first hyperchaotic system [7], marking a major breakthrough in chaos theory. Hyperchaotic systems, defined by multiple positive Lyapunov exponents, exhibit more complex dynamics than conventional chaotic systems [8,9]. These systems, typically of order 4 or higher, have gained interest for their potential use in secure communications due to their strong sensitivity to initial conditions and complex behavior [10–12].

The development of hyperchaotic systems has undergone several stages, including theoretical research [13–15], numerical modeling [16,17], experimental validation [18], and application exploration [19,20]. Building on this foundation, this work introduces a novel 7-dimensional autonomous chaotic system, designed to enhance sensitivity and complexity. The proposed system exhibits three positive Lyapunov exponents, providing a high degree of unpredictability and security, making it highly suitable for cryptographic applications.

The study is structured as follows: Section 2 presents the mathematical formulation of the proposed system. Section 3 analyzes its dynamics using properties such as stability, dissipativity, and bifurcation, along with phase portraits and other key characteristics of the proposed hyperchaotic system. Numerical simulations conducted in Mathematica provide insights into its dynamic behavior, laying the groundwork for its application in secure communications and other technological domains.

2. CONSTRUCTION OF THE NOVEL SEVEN-DIMENSIONAL SYSTEM

The proposed autonomous system can be expressed as follows:

$$\begin{aligned}
\frac{dx_1}{dt} &= -ax_1 + bx_2 + cx_4 - dx_3x_5 - x_3x_7 \\
\frac{dx_2}{dt} &= -ex_2 + fx_1 - gx_5 - hx_1x_3 + cx_3x_4 \\
\frac{dx_3}{dt} &= -ix_3 + dx_5 + gx_7 + x_1x_2 - x_2x_4 \\
\frac{dx_4}{dt} &= -x_4 + x_5 + dx_6 - jx_2x_3 - kx_5x_7 \\
\frac{dx_5}{dt} &= -hx_5 + dx_1 - dx_3 + x_2x_7 + x_6x_7 \\
\frac{dx_6}{dt} &= -lx_6 - x_2 + x_5 + mx_1x_7 - nx_3x_4 \\
\frac{dx_7}{dt} &= -ex_7 + jx_3 - dx_5 + hx_1x_2 + x_4x_5
\end{aligned} \tag{1}$$

The system's positive parameters are denoted as a, b, c, d, e, f, g, h, I, j, k, l, m, and n, while its state variables are given by x1, x2, x3, x4, x5, x6, and x7. When the following system parameter values are selected, the seven-dimensional system (1) exhibits a strange attractor:

$a = 15$, $b = 13$, $c = 0.4$, $d = 0.5$, $e = 14$, $f = 38$, $g = 1.5$, $h = 2$, $I = 6$, $j = 3$, $k = 5$, $l = 15.1$, $m = 30$, and $n = 4$. The initial values are given as follows: $x1(0) = 5, x2(0) = 0.7, x3(0) = 3, x4(0) = 10, x5(0) = 0.2, x6(0) = 0.1$, and $x7(0) = 0.6$.

3. ANALYSIS OF THE NEW HYPERCHAOTIC SYSTEM

This section presents an examination of the novel hyperchaotic system, describing its qualitative features explored in this work.

3.1. System Equilibrium

Equilibrium points of the proposed hyperchaotic system (1) are derived through the solution of nonlinear equations:

$$
\begin{aligned}
0 &= -ax_1 + bx_2 + cx_4 - dx_3x_5 - x_3x_7 \\
0 &= -ex_2 + fx_1 - gx_5 - hx_1x_3 + cx_3x_4 \\
0 &= -ix_3 + dx_5 + gx_7 + x_1x_2 - x_2x_4 \\
0 &= -x_4 + x_5 + dx_6 - jx_2x_3 - kx_5x_7 \\
0 &= -hx_5 + dx_1 - dx_3 + x_2x_7 + x_6x_7 \\
0 &= -lx_6 - x_2 + x_5 + mx_1x_7 - nx_3x_4 \\
0 &= -ex_7 + jx_3 - dx_5 + hx_1x_2 + x_4x_5
\end{aligned} \tag{2}
$$

The single points of equilibrium turn into:

$E0 = \{x1 = 0, x2 = 0, x3 = 0, x4 = 0, x5 = 0, x6 = 0, x7 = 0\}$. Jacobian matrix of system(1),let

$$
f = \begin{cases}
f1 = \frac{dx_1}{dt} = -ax_1 + bx_2 + cx_4 - dx_3x_5 - x_3x_7 \\
f2 = \frac{dx_2}{dt} = -ex_2 + fx_1 - gx_5 - hx_1x_3 + cx_3x_4 \\
f3 = \frac{dx_3}{dt} = -ix_3 + dx_5 + gx_7 + x_1x_2 - x_2x_4 \\
f4 = \frac{dx_4}{dt} = -x_4 + x_5 + dx_6 - jx_2x_3 - \mathrm{k}x_5x_7 \\
f5 = \frac{dx_5}{dt} = -hx_5 + dx_1 - dx_3 + x_2x_7 + x_6x_7 \\
f6 = \frac{dx_6}{dt} = -lx_6 - x_2 + x_5 + mx_1x_7 - nx_3x_4 \\
f7 = \frac{dx_7}{dt} = -ex_7 + jx_3 - dx_5 + hx_1x_2 + x_4x_5
\end{cases} \tag{3}
$$

$$
\mathrm{J} = \begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} & \frac{\partial f_1}{\partial x_5} & \frac{\partial f_1}{\partial x_6} & \frac{\partial f_1}{\partial x_7} \\
\frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} & \frac{\partial f_2}{\partial x_5} & \frac{\partial f_2}{\partial x_6} & \frac{\partial f_2}{\partial x_7} \\
\frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} & \frac{\partial f_3}{\partial x_5} & \frac{\partial f_3}{\partial x_6} & \frac{\partial f_3}{\partial x_7} \\
\frac{\partial x_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} & \frac{\partial f_4}{\partial x_5} & \frac{\partial f_4}{\partial x_6} & \frac{\partial f_4}{\partial x_7} \\
\frac{\partial f_5}{\partial x_1} & \frac{\partial f_5}{\partial x_2} & \frac{\partial f_5}{\partial x_3} & \frac{\partial f_5}{\partial x_4} & \frac{\partial f_5}{\partial x_5} & \frac{\partial f_5}{\partial x_6} & \frac{\partial f_5}{\partial x_7} \\
\frac{\partial f_6}{\partial x_1} & \frac{\partial f_6}{\partial x_2} & \frac{\partial f_6}{\partial x_3} & \frac{\partial f_6}{\partial x_4} & \frac{\partial f_6}{\partial x_5} & \frac{\partial f_6}{\partial x_6} & \frac{\partial f_6}{\partial x_7} \\
\frac{\partial f_7}{\partial x_1} & \frac{\partial f_7}{\partial x_2} & \frac{\partial f_7}{\partial x_3} & \frac{\partial f_7}{\partial x_4} & \frac{\partial f_7}{\partial x_5} & \frac{\partial f_7}{\partial x_6} & \frac{\partial f_7}{\partial x_7}
\end{bmatrix} \tag{4}
$$

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$$J = \begin{bmatrix} -a & b & -dx_5 - x_7 & c & -dx_3 & 0 & -x_3 \\ f - hx_3 & -e & -hx_1 - x_4 & -cx_3 & -g & 0 & 0 \\ x_2 & x_1 - x_4 & -i & -x_2 & d & 0 & g \\ 0 & -jx_3 & -jx_2 & -1 & 1 - kx_7 & d & -kx_5 \\ d & x_7 & -d & 0 & -h & x_7 & x_2 + x_6 \\ mx_7 & -1 & -nx_4 & -nx_3 & 1 & -l & mx_7 \\ hx_2 & hx_1 & j & x_5 & -d + x_4 & 0 & -e \end{bmatrix} \quad (5)$$

For the point of equilibrium:

$E0 = \{x1 = 0, x2 = 0, x3 = 0, x4 = 0, x5 = 0, x6 = 0, x7 = 0\}$, with values $a = 15, b = 13, c = 0.4, d = 0.5, e = 14, f = 38, g = 1.5, h = 2, I = 6, j = 3, k = 5, l = 15.1, m = 30$, and $n = 4$.

The following is the outcome of the Jacobian matrix:

$$J_0 = \begin{bmatrix} -15 & 13 & 0 & 0.4 & 0 & 0 & 0 \\ 38 & -14 & 0 & 0 & -1.5 & 0 & 0 \\ 0 & 0 & -6 & 0 & 0.5 & 0 & 1.5 \\ 0 & 0 & 0 & -1 & 1 & 0.5 & 0 \\ 0.5 & 0 & -0.5 & 0 & -2 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & -15.1 & 0 \\ 0 & 0 & 3 & 0 & -0.5 & 0 & -14 \end{bmatrix} \quad (6)$$

To obtain its eigenvalues, we set $|\lambda I - J0| = 0$. The resulting eigenvalues equivalent with the stable point E0 (0, 0, 0, 0, 0, 0, 0), are calculated as shown below:

$\lambda_1 = -36.7377, \quad \lambda_2 = -15.011, \quad \lambda_3 = -14.5232,$

$\lambda_4 = -5.41581, \quad \lambda_5 = -2.02482, \quad \lambda_6 = -1.00679,$

$\lambda_7 = 7.7095$

The results show that the system (1), at the equilibrium point E0, has both positive and negative real eigenvalues. Therefore, this equilibrium point is unstable.

3.2. Dissipativity

The novel system (1) can be expressed in vector notation as:

$$f = \begin{cases} f_1 = \frac{dx_1}{dt} = -ax_1 + bx_2 + cx_4 - dx_3x_5 - x_3x_7 \\ f_2 = \frac{dx_2}{dt} = -ex_2 + fx_1 - gx_5 - hx_1x_3 + cx_3x_4 \\ f_3 = \frac{dx_3}{dt} = -ix_3 + dx_5 + gx_7 + x_1x_2 - x_2x_4 \\ f_4 = \frac{dx_4}{dt} = -x_4 + x_5 + dx_6 - jx_2x_3 - kx_5x_7 \\ f_5 = \frac{dx_5}{dt} = -hx_5 + dx_1 - dx_3 + x_2x_7 + x_6x_7 \\ f_6 = \frac{dx_6}{dt} = -lx_6 - x_2 + x_5 + mx_1x_7 - nx_3x_4 \\ f_7 = \frac{dx_7}{dt} = -ex_7 + jx_3 - dx_5 + hx_1x_2 + x_4x_5 \end{cases} \quad (7)$$

The vector field f's divergence in $\mathbf{R}^7$ is written as:

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$$\nabla \cdot \mathrm{f} = \frac{\partial \mathrm{f}_1}{\partial \mathrm{x}_1} + \frac{\partial \mathrm{f}_2}{\partial \mathrm{x}_2} + \frac{\partial \mathrm{f}_3}{\partial \mathrm{x}_3} + \frac{\partial \mathrm{f}_4}{\partial \mathrm{x}_4} + \frac{\partial \mathrm{f}_5}{\partial \mathrm{x}_5} + \frac{\partial \mathrm{f}_6}{\partial \mathrm{x}_6} + \frac{\partial \mathrm{f}_7}{\partial \mathrm{x}_7} \quad (8)$$

It should be noted that the rate at which volumes change under the flow Φ_t generated by f is measured by $\nabla \cdot f$. let $\boldsymbol{D}$ be a region in $\mathbf{R}^7$ with a smooth boundary. Next, examine the image of $\boldsymbol{D}$ corresponding to Φ_t, the time t of the flow of f, where $\boldsymbol{D(t)} = \Phi_t(t)$. In this case, $V(t)$ represents the volume of $D(t)$. According to Liouville's theorem [55,67], we obtain the following:

$$\frac{dV}{dt} = \int_{D(t)} (\nabla \cdot f) dx_1 dx_2 dx_3 dx_4 dx_5 dx_6 dx_7 (9)$$

For the system (1), we find that:

$$\nabla \cdot f = -(a + e + i + 1 + h + l + e) < 0 \quad (10)$$

Because a, e, i, h and l are positive constants, substituting Equation (10) into Equation (9) and simplifying, we get:

$$\frac{dV}{dt} = -(a + e + i + 1 + h + l + e) \int_{D(t)} dx_1 dx_2 dx_3 dx_4 dx_5 dx_6 \quad (11)$$

$$\frac{dV}{dt} = -(a + e + i + 1 + h + l + e) V(t)$$

$$= e^{-65.1t} V(t)$$

The unique solution to the first-order linear differential equation (11) is derived as follows:

$$\mathrm{V(t)} = \mathrm{V(0)e^{-(a+e+i+1+h+l+e)t}}$$

$$= \mathrm{V(0)e^{-65.1t}} \quad (12)$$

According to Equation (12), any volume V(t) decreases exponentially to zero over time. Consequently, equations (1) describe a dissipative dynamical system.

3.3. Lyapunov Dimensions and Exponents

A chaotic system relies on sensitivity to initial conditions, which the Lyapunov Exponents measure in accordance with nonlinear dynamical theory. A Lyapunov exponent represents the average rate at which two nearby trajectories diverge (or converge) over time. This sensitivity highlights the unpredictable nature of chaotic systems and emphasizes their complex dynamical behavior.

Furthermore, seven Lyapunov exponents of the nonlinear dynamical system (1), with specified parameters, were calculated as follows:

$$LE1 = 6.68949, LE2 = 2.65441, LE3 = 1.4784, LE4 = -8.79264, LE5 = -12.8942,$$

$$LE6 = -14.8192, LE7 = -39.706$$

As shown, this system exhibits chaotic behavior because its largest Lyapunov exponent is positive. This indicates that trajectories in the phase space diverge exponentially, a hallmark of chaos. Since the remaining four Lyapunov exponents are negative, the system shows a stable contraction along those dimensions. Additionally, LE_1, LE_2, *and* LE_3 are positive Lyapunov exponents, highlighting the system's complexity. The system is therefore classified as hyperchaotic due to having more than one positive Lyapunov exponent.

Another common feature of chaotic systems is the fractal dimension, which provides insight into their geometric complexity. The fractal dimension is derived from the Lyapunov exponents and is used to calculate the Kaplan–Yorke dimension. The Kaplan-Yorke dimension, denoted as D_{KY}, is given by [15]:

$$\mathrm{D_{KY}} = \mathrm{j} + \frac{1}{|\mathrm{L_{Ej+1}}|}\sum_{\mathrm{i=1}}^{\mathrm{j}} \mathrm{LE_i} \quad (13)$$

Here, the first j Lyapunov exponents are non-negative, as indicated by j, which is the greatest value of i that simultaneously satisfies $\sum_{i=1}^{j} LE_i > 0$ and $\sum_{i=1}^{j+1} LE_i < 0$. Based on the Lyapunov exponents sequence, $\mathrm{LE_i}$ is in declining order of the sequence. The upper limit of the system information's dimension is denoted by D_{KY}.

Since $LE_1 + LE_2 + LE_3 + LE_4 > 0$ *and* $LE_1 + LE_2 + LE_3 + LE_4 + LE_5 + LE_6 + LE_7 < 0$, The Kaplan-Yorke dimension for the system in this work can be obtained by analyzing the seven calculated Lyapunov exponents presented above, which show that j equals four. Concerning the new chaotic system, the Lyapunov dimension is:

$$\begin{aligned}
\mathrm{D_{KY}} &= \mathrm{j} + \frac{1}{|\mathrm{LE_{j+1}}|}\sum_{\mathrm{i=1}}^{\mathrm{j}} \mathrm{LE_i} \\
\mathrm{D_{KY}} &= 4 + \frac{1}{|\mathrm{LE_{j+1}}|}\sum_{\mathrm{i=1}}^{4} \mathrm{LE_i} = 5 + \frac{\mathrm{LE_1+LE_2+LE_3+LE_4}}{|\mathrm{LE_5}|} \\
&= 4 + \frac{(6.68949+2.65441+1.4784-8.79264)}{12.8942} \\
&= 4.15741
\end{aligned}$$

This result suggests that the Lyapunov dimension of the system (1) is fractional. The system exhibits aperiodic trajectories due to its fractal nature, and its neighboring trajectories diverge as well. Therefore, this nonlinear system can be considered genuinely chaotic.

3.4. Phase Portraits

Using the *MATHEMATICA* program, the numerical simulation has been completed to analyze the chaotic behavior of this nonlinear system. This system demonstrates intricate and abundant chaotic dynamics, which strange attractors characterize. These attractors are visualized in three-dimensional space in Figures (1–3), and in two-dimensional projections in Figures (4–6). The system's dynamics reveal complex patterns, with the attractors exhibiting irregular yet structured behavior, typical of chaotic systems. The topology observed in the phase portraits resembles the so-called "butterfly effect." As depicted in Figure 1, this structure resembles the contours of a butterfly in flight, illustrating the sensitivity of chaotic systems to initial conditions. The "butterfly effect" emphasizes how small changes in initial conditions can lead to vastly different outcomes—a defining characteristic of chaotic systems.

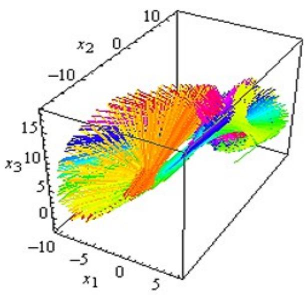

Figure 1. 3D Representation of Chaotic Attractors (x_1-x_2-x_3)

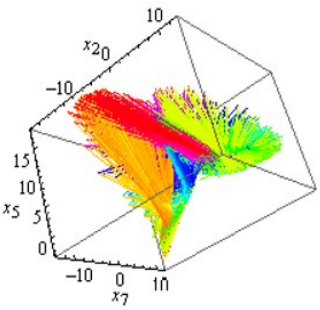

Figure 2. 3D Representation of Chaotic Attractors (x_2-x_5-x_7)

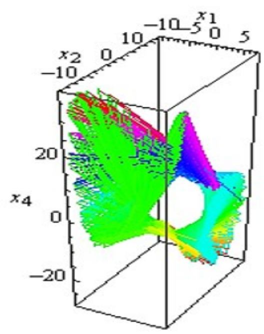

Figure 3. 3D Representation of Chaotic Attractors (x_1-x_2-x_4)

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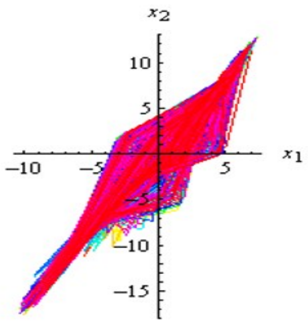

Figure 4. 2D Representation of Chaotic Attractors (x_2- x_1)

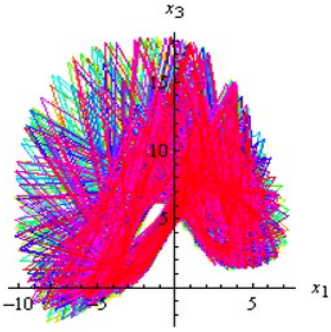

Figure 5. 2D Representation of Chaotic Attractors (x_3 -x_1)

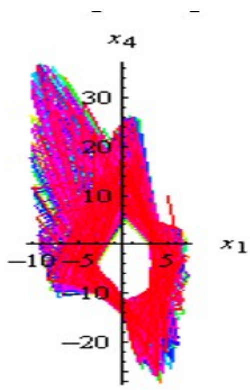

Figure 6. 2D Representation of Chaotic Attractors (x_4- x_1)

3.5. Waveform Analysis in a New Chaotic System

It is well known that a chaotic system typically exhibits aperiodic waveforms. To illustrate the chaotic nature of the suggested system. time-versus-state plots obtained from *MATHEMATICA* simulations are shown in Figures 7–13. Figures (7–13) display the time-domain waveforms of $x_1(t), x_2(t), x_3(t), x_4(t), x_5(t), x_6(t), x_7(t)$. Each of these waveforms is aperiodic, confirming the system's chaotic nature. It is possible to distinguish between chaotic motion and multiple periodic motions, which can both exhibit complex behavior, by looking for non-cyclical features in the time domain waveform.

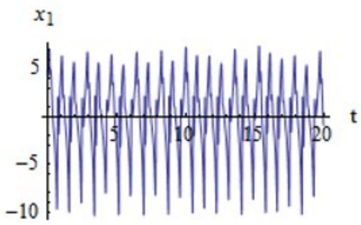

Figure 7. Time vs. x_1 in a novel chaotic system

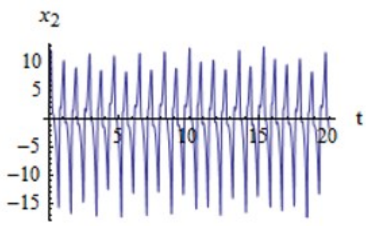

Figure 8. Time vs. x_2 in a novel chaotic system

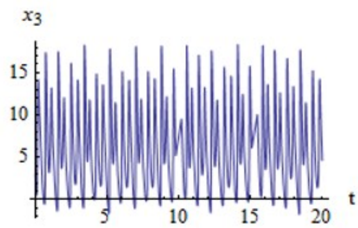

Figure 9. Time vs. x_3 in a novel chaotic system

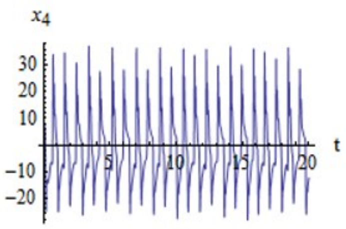

Figure 10. Time vs. x_4 in a novel chaotic system

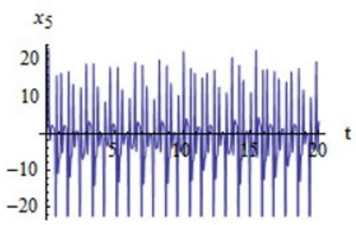

Figure 11. Time vs. x_5 in a novel chaotic system

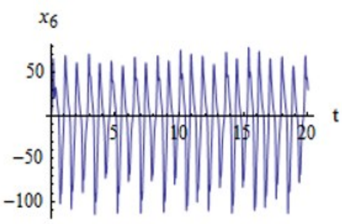

Figure 12. Time vs. x_6 in a novel chaotic system

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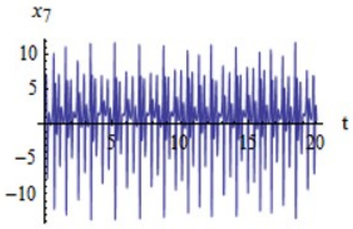

Figure 13. Time vs. x_7 in a novel chaotic system

3.6. Initial condition sensitivity

The long-term unpredictability of a chaotic system is arguably its most defining quality. This intrinsic property arises from the system's solutions being highly sensitive to initial conditions. Even the slightest differences between two starting conditions — regardless of how similar they may appear—will eventually cause the trajectories to become vastly separated over time. This sensitivity leads to a situation where, in the future, it becomes increasingly complex to make accurate predictions about the system's state. Any attempt to predict the system's behavior is constrained by the precision of the initial conditions. As shown in Figures 16–22, even slight variations in initial conditions result in dramatically different trajectories. These figures demonstrate the significant impact that initial conditions have on the system's chaotic behavior. The initial conditions used in the simulations are as follows:

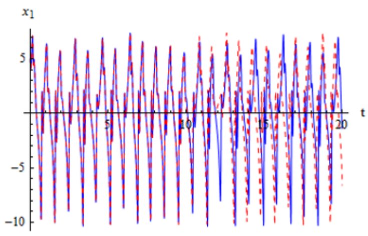

Figure 14 $x_1(t)$ Sensitivity Analysis of the Novel System

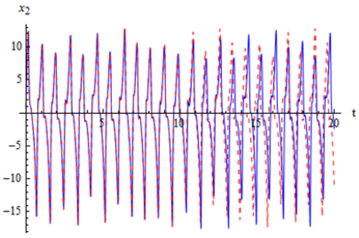

Figure 15. $x_2(t)$ Sensitivity Analysis of the Novel System

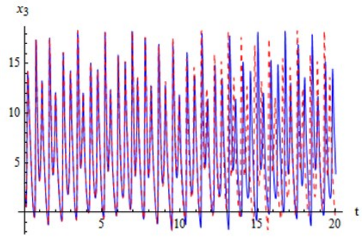

Figure 16. $x_3(t)$ Sensitivity Analysis of the Novel System

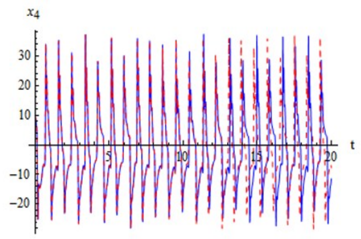

Figure 17. $x_4(t)$ Sensitivity Analysis of the Novel System

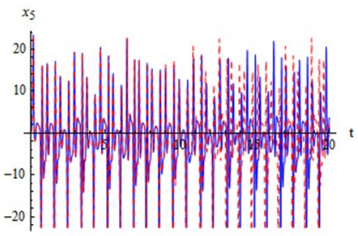

Figure 18. $x_5(t)$ Sensitivity Analysis of the Novel System

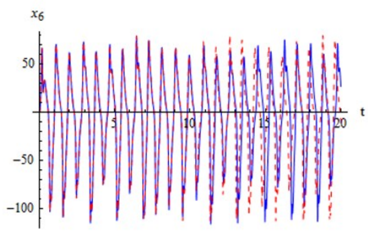

Figure 19. $x_6(t)$ Sensitivity Analysis of the Novel System

https://doi.org/10.31436/iiumej.v26i3.3590

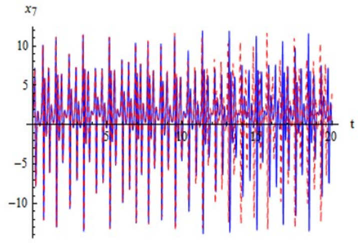

Figure 20. $x_7(t)$ Sensitivity Analysis of the Novel System

3.7. Bifurcation Diagram

The *MATHEMATICA* software was used to simulate the new system (1) and determine the maximum value of the variable x_1. By varying the values of the parameter *a*, we observe the intriguing region of x_1 where the system changes its dynamics and values of x_1 begin to bifurcate. This bifurcation phenomenon is one of the most prominent characteristics of chaotic systems and clearly indicates the system's sensitivity to parameter variation. This behavior is particularly evident in the range *a* = 14.2 and *a* = 14.4, where the system undergoes a transition to more complex dynamics. The bifurcation diagram provides insight into how the system evolves and displays its chaotic dynamics. Figure 23 illustrates how bifurcation occurs in the new system, showcasing the dramatic changes in its properties as the parameter *a* is adjusted.

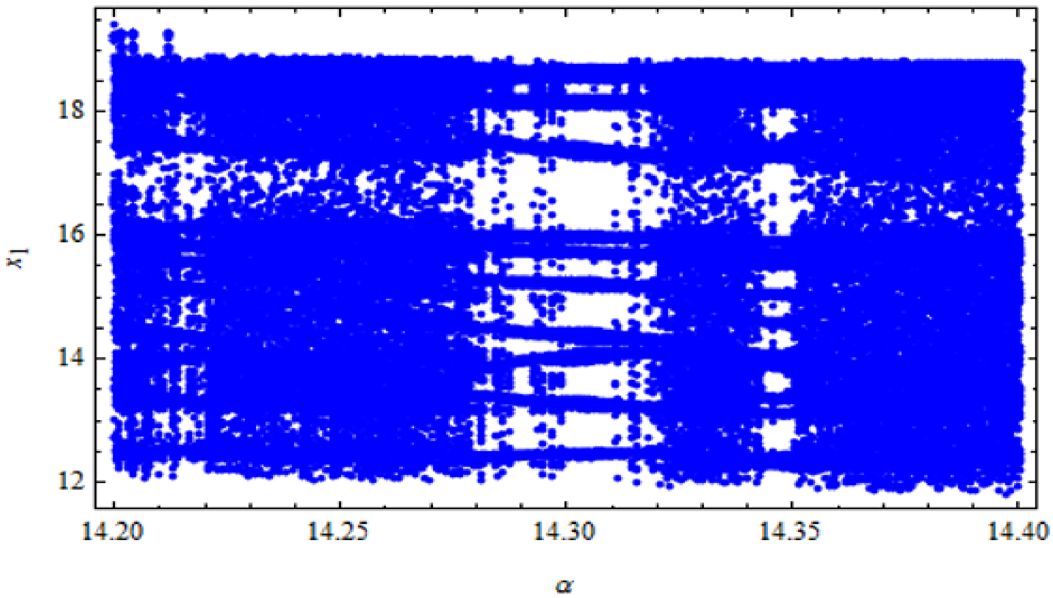

Figure 21. Bifurcation diagram of the variable x_1 for increasing ***a***

3.8. Analysis of Key Spaces

The total number of distinct keys that can be used throughout the encryption process is referred to as the *keyspace* size. A strong encryption algorithm should be sensitive to secret keys.

The *keyspace* should be large enough to prevent attacks using brute force. Generally speaking, a smaller *keyspace* makes the system more vulnerable to attacks. Cryptographically, to make brute force attacks ineffective, the size of the *keyspace* should be at least 2^{128} Since the parameters $a, b, c, d, e, f, g, h, i, j, k, l, m,$ and n and initial conditions $x_1(0), x_2(0), x_3(0), x_4(0), x_5(0), x_6(0)$, and $x_7(0)$ serve as the hidden keys in this algorithm and used as secret key. In this instance, the *keyspace* size can reach $(10^{14})^{21} = 10^{294} \simeq 2^{970}$, assuming a precision of 10^{-14}, which is significantly larger than 2^{128}. Therefore, the *keyspace* is sufficiently large to withstand the brute-force attacks.

4. CONCLUSION

This paper introduces a new seven-dimensional (7D) chaotic system. The system exhibits intricate behavior and high sensitivity to initial conditions, showcasing its potential for advanced applications in the field of chaos theory. Through thorough analyses, including the Kaplan-Yorke dimension and equilibrium point evaluations, we demonstrate the system's strongly chaotic nature. This is reflected in its three positive Lyapunov exponents, which confirm its hyperchaotic nature and its ability to generate a wide variety of strange attractors.

The system's dissipative nature and inherent instability further highlight its suitability for critical applications in secure communication and encryption. In such contexts, sensitivity to initial conditions and dynamic complexity is an essential characteristic. These attributes enable the system to effectively obscure data, making it robust against various types of cryptographic attacks, including brute-force attacks, as demonstrated by the findings of this study.

Additionally, numerical simulations conducted using Mathematica provided a detailed exploration of the system's dynamics, including its phase diagrams and chaotic trajectories. These simulations have been instrumental in validating the theoretical properties of the system and in illustrating its potential for real-world applications.

Overall, the proposed seven-dimensional framework marks a significant advancement in the study of chaotic systems and their applications. It provides a solid foundation for future research, particularly in the development of innovative encryption algorithms and emerging security technologies. Subsequent investigations will focus on its broader applicability and further optimization for real-world cryptographic systems, ensuring its viability in the evolving landscape of secure communication.

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