AN INVESTIGATION ON SOFT MAGNETIC AND NON-MAGNETIC MATERIALS UNDER LOW FREQUENCY

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ABSTRACT: The recent development of magnetic sensors in biomedical sector has called for urgent investigation on the characterization and relative magnetism of magnetic and nonmagnetic materials. This paper proposes a novel technique to categorize the magnetic and non-magnetic materials by obtaining their impedance peaks. The magnetization effect of a magneto-inductive sensor was detected in low frequency range for different magnetic and nonmagnetic core materials. The distinctive impedance peak values have been obtained for each materials used as core which result in showing different magneto-inductive effect. The results have shown a dominant behaviour of magnetic materials over non-magnetic materials due to the variation of permeability and magnetic strength (number of turns). Moreover, variations in permeability originated by the applied alternative current field results in changing the depth (insertion depth) and also in the impedance. The influence of the permeability on materials with different frequencies is detailed. Also, concentration is paid to the recent studies in the application base. It should be mentioned that high impedance peaks mean high output voltage in the secondary side. The novelty of this technique lies in the simplicity of ordinary circuits which are used in the experimental setup for characterization purpose. As a whole, this paper aims to investigate and characterize magnetic and non-magnetic materials; thus opening up a new branch of research for the application of soft magnetic materials in biomedical field.

ABSTRAK: Pembangunan penyelidikan terhadap bahan bermagnet dan bukan bermagnet yang mempunyai karakteristik dan kemagnetan relatif telah dijalankan dengan intensif untuk menghasilkan sensor magnet bagi sektor biomedik. Kajian ini adalah untuk mencadangkan teknik bagi menketagorikan bahan bermagnet dan bukan bermagnet dengan mendapatkan puncak rintangan bahan tersebut. Sensor bahan teraruh magnet mengesan frekuensi yang rendah kemagnetan daripada teras bahan bermagnet dan bukan bermagnet. Puncak rintangan yang signifikan terhasil daripada bahan tersebut menunjukkan kedominan bahan bermagnet daripada bahan bukan bermagnet. Ini adalah kerana variasi dan kekuatan tarikan kemagnetan (pengaruh bilangan pusingan). Variasi kemagnetan terhasil daripada penggunaan alternatif medan bahan yang menyumbang kepada perubahan kekuatan kemagnetan (kedalaman pergerakan bahan) dan kerintangannya. Pengaruh kemagnetan bahan dan frekuensi yang dihasilkan dijelaskan dengan lanjut dalam laporan ini. Diketahui bahawa peningkatan kerintangan akan mengakibatkan peningkatan voltan keluar dalam bahagian sekunder. Kelainan teknik ini terletak pada penggunaan litar yang mudah dipasang dan digunakan dalam pengkonstrukturan experimen bagi mendapatkan karekter kemagnetan bahan. Kesimpulannya, kajian ini adalah untuk menyiasat karekteristik bahan bermagnet dan bukan bermagnet untuk digunakan dalam usaha penyelidikan mencari bahan dengan medan magnet yang sesuai dan boleh diaplikasikan dalam bidang biomedik.

KEYWORDS: *inductive transducer; magneto-inductive; inductive element under test (IEUT);, resistive material under test (MUT)*

1. INTRODUCTION

Magnetic materials are of essential importance in many industrial, biomedical and engineering applications. Over the past years research and studies have shown significant development for its numerous range of applications, especially in the biomedical sector. Today's dynamic expression of research interest has called for better understanding of the properties of soft magnetic and non-magnetic materials. The reason for the extensive use of magnetic materials in biomedical field is that they can be easily manipulated by an external magnetic field. Recently researchers have focused also on new technologies like nanostructured magnetic materials in order to characterize and specify magnetic materials and its properties for suitable applications. Magnetic materials are usually used in non-contact inductive sensors in the form of core materials. Magnetic materials can also be employed for magnetic resonance imaging (MRI), drug and gene delivery, cell separation, invasive surgery, radionuclide therapy, hyperthermia and other applications [1-6] where wiring accessibility may cause infection and it does not sound a sensible possibility in harsh conditions. Thus, contactless transmission of data signal and power can offer a solution alternative to wiring connectivity. Today non-contact sensor medical experts are able to intelligently monitor medical condition of patients like detecting and monitoring the respiratory system, the contraction of blood vessels, cardiac pressure disorders etc.

A sensor such as the heart rate sensor takes an electrocardiogram reading from the heart rate of a patient and the sensor then relays the collected data to a computer, where the patient is monitored concurrently. Moreover, wireless power transmission system uses an inductive coupling (non-contact) means to supply energy to the implanted devices within the human body that eliminates the need for surgery for battery replacement, thus making it safer for patients from getting infected. In this type of wireless system, noncontact inductive sensors use different type of magnetic material. The development of various sensors has opened up several possibilities for significant improvements in the advancement of bio-implantable devices or medical equipment. Hence many problems regarding magnetic properties of the magnetic materials exist to be examined from the elementary viewpoint. To select the magnetic material and be completely familiar with the characterization of its magnetic properties proper investigation is required. In biomedical implants like aneurysm clip, the relative amount of magnetism should be evaluated. Aneurysm clip is an implant used for closing blood vessels located in hardly accessible regions of patients' body. It generally undergoes magnetic resonance (MR) procedures in MR systems. Previous research [7] of this particular implant emphasized on the significance in evaluating the relative amount of magnetism present in the aneurysm clip. It is found that only the patients or individuals with non-ferromagnetic or weakly ferromagnetic clips are allowed into the MR environment. Hereby the identification of materials appears to guide in using for suitable applications. Extensive studies [8-11] have gained interest on various magnetic materials for its' wide range of application. Especially magneto-inductive effect on soft ferromagnetic materials as well as ferromagnetic materials has been observed by researchers which led to the design of various noncontact sensors over the years. Under an ac magnetic field excitation, these sensors resonate at their resonant frequencies. In the magneto-inductive sensor, the alternating current (AC) impedance of a soft ferromagnetic material alters with the longitudinal component of a direct current (dc) magnetic field. The relative amount of the magnetic materials and the characteristics of these material properties should be taken into consideration. In this paper, we have emphasized on a technique to characterize the magnetic and nonmagnetic materials and as a part of experimental work four materials were selected. This method can be used in case of various noncontact inductive sensors including above mentioned wireless passive sensors as both the sensing and biasing elements of these sensors are magnetic materials. This work proposes a novel technique for investigation of soft magnetic and non-magnetic materials under low frequency using experimental approach. The proposed work aims at identifying proper magnetic material to be used as core of inductive sensory transducers, ultimately identifying them for sensors fabrication, thus differentiating the materials by gaining distinctive impedance peaks with unique impedance values. Design specifications of the proposed model includes the sensing circuit and simulation results. Moreover, we have characterized magnetic and non-magnetic materials for biomedical applications which require low frequency range.

2. THEORY OF WIRELESS ENERGY SYSTEM WHERE MAGNETO-INDUCTIVE SENSORS ARE USED

The wireless energy systembasically concentrates on transferring the data and required energy to a small terminal via noncontact means. Inductive coupling has been preferred as the most effective means of transferring energy to low power implanted devices. The magnetic energy transferring techniqueis less sensitive to its surroundings which do not interact with the coupled magnetic field. Moreover, the magnetic fields do not impact on human physically [12]. The inductive coupling can be divided into two parts; primary and secondary (two air-core coils) to transfer power wirelessly. The concept is simple: to use the resonance phenomena to wirelessly transmit power. The inductive coupling system acts as magneto-inductive sensor where magnetic and non-magnetic materials are inserted in the form of core. Magneto-inductive sensor is basically represented by a hollow solenoid, wounded with core made of the material under test as illustrated in Fig.1; the solenoid consists of primary coil and secondary coil. When a core material under test is moved in and out continuously at a given frequency, it leads to producing a change in the inductance and hence a change in the corresponding output. The output from the circuit is calibrated directly against the value of the input such that it becomes a representative of the material the core is made of. The inductance to be measured is dependent on the nature of materials (μr).

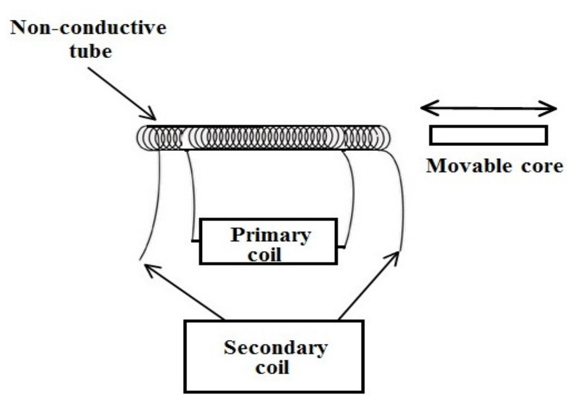

Fig. 1: Material testing coil.

Changes made in the impedance of the test coil are made to be reflected as changes of the inductance of the sensor circuit and magnetically coupled to the test coil. Here the distance changes between the readout circuit and the sensing circuit that are related to the leakage flux lines. The inductance to be measured is dependent on the nature of material (μr).

3. CIRCUIT ANALYSIS

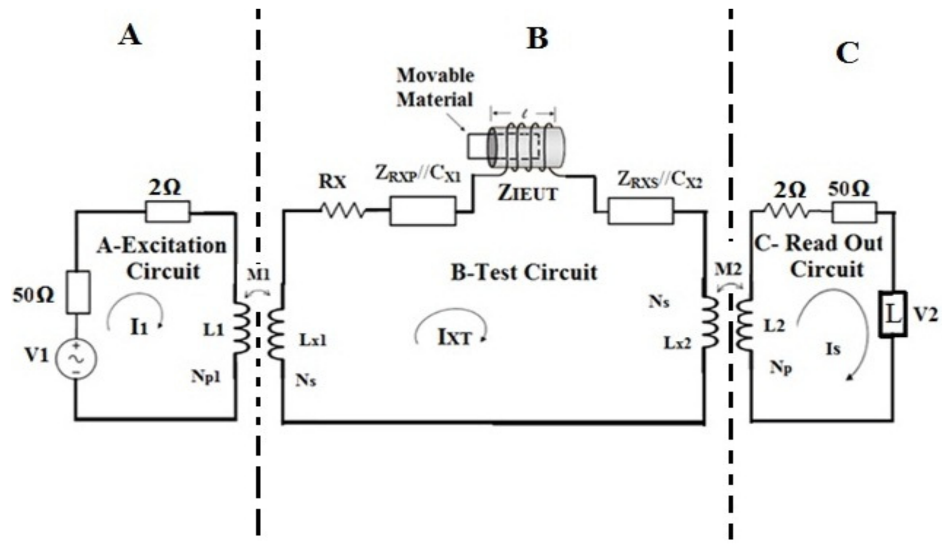

Fig. 2: Schematic representation of the circuit diagram.

Figure 2 is used for obtaining the theoretical and experimental results in this work. This model circuit is integrated with three inductive circuits: excitation circuit (marked A), test circuit (marked B), and read-out circuit (marked C). This whole setup makes up a non-contact method of characterizing materials, which is based on inductive coupling for energizing the Inductive Element Under Test/ Resistive Material under Test with cores made up of the materials being characterized (see Fig.3). A set of two experiments were carried out, the first experiment was done with Inductor for two set of coils, and the second experiment was done by replacing the inductor with resistor of two different resistive values. A series of graph was plotted, and then analysed based on their unique peak values. Vin is set at a frequency of 1.0 kHz and the supply voltage is set at 3V.

a. Excitation circuit provides an external power supply, which is connecting to the test circuit by non-contact means that is through inductive coupling. There exists a mutual inductance M_1 between the loops. The signal source circuit has an impedance of $Z_1 = R_1 + jwL_1$
b. Test circuit is the transducer processing circuit, connected to both the excitation circuit and the read-out circuit through inductive coupling. To enable the measurement without direct electrical contact, two inductive couplings are used $(L, L_1 and L_2)$. M_1 is the mutual inductance which exists between the excitation circuit and the test circuit, and M_2 is the mutual inductance existing between the test circuit and the read-out circuit. $Z_{RXP} \parallel C_{X1}$ and $Z_{RXS} \parallel C_{X2}$ is the parallel

impedance of the fixed resistor R and the bypass capacitor C_{X1} and C_{X2} respectively. C_{X1} and C_{X2} serves as the low impedance bypass capacitors for the coupled AC signal. The transducer processing circuit, has a transducer impedance Z_{IEUT}, self-impedance $R_X + j\omega L$, and reflected impedances Z_{RXP} and Z_{RXS}.

c. Readout circuit is the signal measuring circuit which measures V_2, the resultant signal voltage produced when a material is inserted in the IEUT coil.

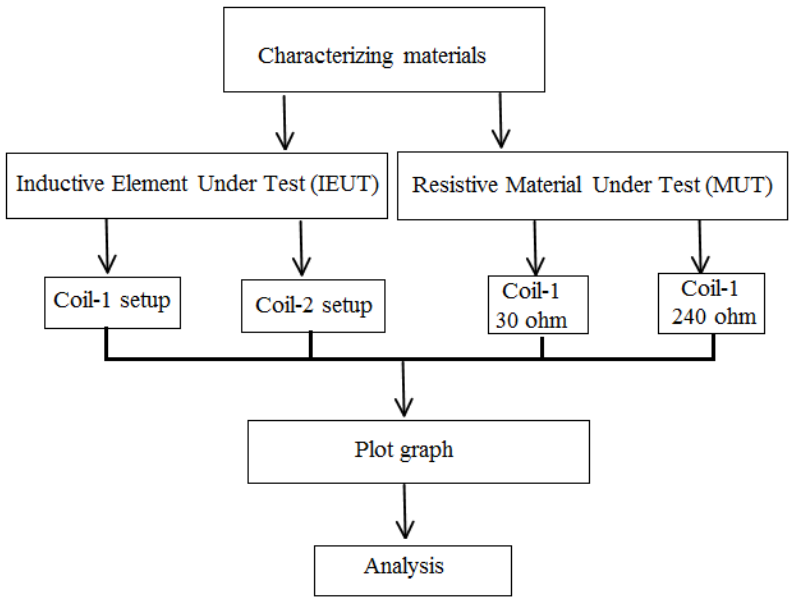

Fig. 3: Flowchart of the experimental setup.

From Fig.2, Z_{IEUT} is derived as a function of change in inductance where Z_{IEUT} is given by Eq. (4)

$$I_{XT} = \frac{V_{MXP}}{\left(\left(j\omega L + R_X\right) + \left(Z_{RXP} \parallel C_{X1}\right) + \left(Z_{RXS} \parallel C_{X2}\right) + Z_{IEUT}\right) + \left(Z_{RXP} + Z_{RXS}\right)} \quad (1)$$

$$Z_{setup} = Z_{RXP} + Z_{RXS} + \left(j\omega L + R_X\right) + \left(Z_{RXP} \parallel C_{X1}\right) + \left(Z_{RXS} \parallel C_{X2}\right) \quad (2)$$

$$I_{XT} = \frac{V_{MXP}}{Z_{setup} + Z_{IEUT}} \quad (3)$$

Hence,

$$Z_{IEUT} = \frac{V_{MXP}}{I_{XT}} - Z_{setup} \quad (4)$$

4. EXPERIMENTAL SETUP

The technique was implemented based on the experimental setup. The equipment used for both the experimental setup includes function generator, oscilloscope, volt-meters and cored coils. The function generator provides the external power needed to excite the primary coil. In the experiment, V_1 was set with an input of $V_{pp} = 3V$ with frequency ranging from 1 kHz to 45 kHz. To obtain experimental results, one of the coils was excited by a signal at a given frequency, which was received and measured through another coil. Measurements were recorded across the resulting coupled signal in the circuit loop from the read-out circuit. The output value V_2 was recorded for every frequency increment of 1 kHz. The inductive transducer coil (IEUT/MUT) is the subject under test. Several different materials were used for the test; steel, plastic and air/ paper core (used as a reference). Two sets of experiment were carried out under similar experimental procedures with two different sets of coil. The electrical specification of the coils used in these two experiments is given in Table 1 and Table 2.

Table 1**:** Specification of primary and secondary coils.

	Number of turns	
	Coil set-1	**Coil set-2**
Primary coil 1	200	400
Primary coil 2	200	400
Secondary coil	1000	2000
Internal diameter (mm):10		
Height (mm):40		
Material of wire: copper		
Thickness of wire (mm): 0.19		

Table 2**:** Model parameter of the circuit.

Symbol	**Description**	**Value**	**Unit**
R_x	Fixed resistor	2	Ω
R_1	Series impedance in the primary coil	50	Ω
R_2	Series impedance in the secondary coil	50	Ω
C_1	Bypass capacitor	220	nF
C_2	Bypass capacitor	220	nF

5. RESULTS AND DISCUSSION

Cores of different materials with the same physical dimensions were used in this work; hence the strength of the magnet will vary in accordance with the core being used. The variation in the strength is due to the varying number of flux lines passing through the core. The permeability of the material μ_r is a measure for the strength of materials. The permeability of non-magnetic materials, such as plastic, aluminium and air is approximately the same as that of free space μ_o. Materials having permeability greater than μ_o are ferromagnetic (soft magnetic), such as steel. The results obtained are discussed in detail; Figures 4-7 present the results obtained by conducting the first experiment for IEUT. The coil specifications used are as stated in Table 1 (Coil-1). Figure 4 shows the inductive peak plotted for air core when no material is inserted in the solenoid coil. A highest peak in impedance (8 kHz, 1.402921605 Ω) is achieved because there was no interruption in the electromagnetic field of the coil. Figure 5 shows the results for plastic core. It exhibits non-magnetic and non-conductive properties similar to air, causing impedance peaks at a frequency similar to that of air (8 kHz, 0.894600647 Ω). The lower

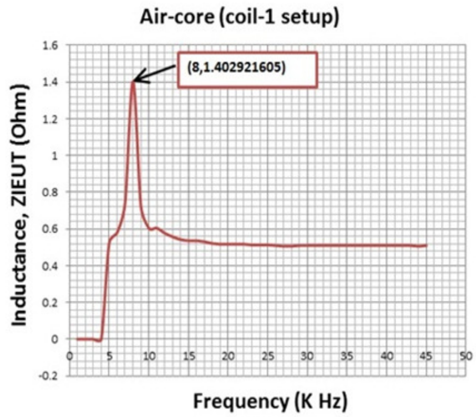

Fig. 4: Inductive peak against frequency variation for air-core coil-1 setup.

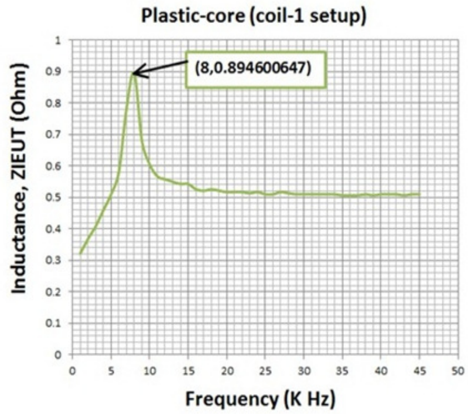

Fig. 5: Inductive peak against frequency variation for plastic-core coil-1 setup.

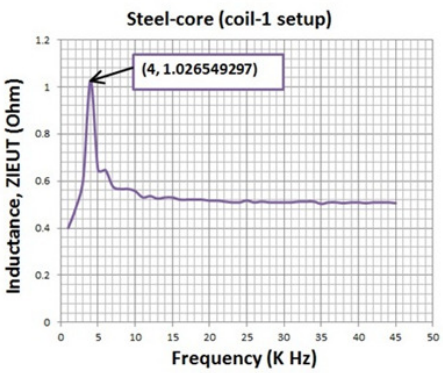

Fig. 6: Inductive peak against frequency variation for steel-core coil-1 setup.

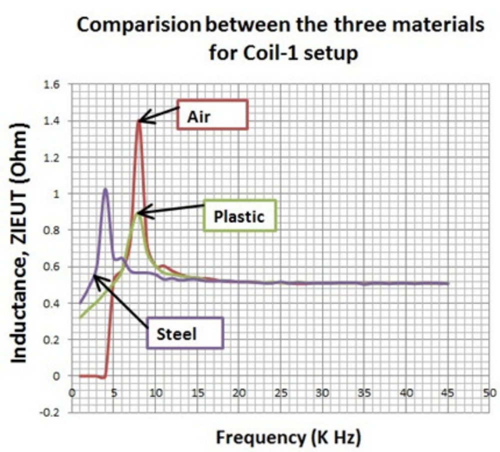

Fig. 7: Overall performance for the three materials with coil-1setup.

permeability value of plastic compared to that of air contributes to lower inductive reactance amplitude peak. The plot in case of steel is presented in Fig. 6 where a decrease in peak value (4 kHz, 1.026549297 Ω) compared to air-core and a plastic-core plot is observed. Generally steel is a good conductor and is also considered as a magnetic material. A core of magnetic material such as steel when placed in a coil or transformer provides a better path than air or plastic for magnetic flux, thereby increasing the inductance of the coil and raising the coupling between windings accordingly. This in turn causes the impedance peaks at a lower value of frequency considering the circuit component value remaining the same. Accordingly the lower peak value is attributed to a lower value of permeability, assuming all other inductive reactance causing reliant factors to remain the same. Figure 7 illustrates the overall performance for the three materials used. Based on the results obtained, the materials tested appeared to exhibit distinctive and unique features of impedance peaks with varying amplitude and frequency, thus characterizing the materials under observation. Among the three materials tested so far, steel was found to be more promising that air or plastic when it comes to applications such as transducers. However, space and weight requirement priorities favour the choice of nonmagnetic materials when the sensitivity is not a pivotal issue of concern. However, in an application-specific situation, conductive material proves better than the non-conductive materials when used as a core for magneto sensor application.

In line with the second experiment, the second set of coil was used with an increased number of primary and secondary turns, as specified in Table 1 (Coil-2). Figure 8-11 show the effects on the inductive peaks by the increased number of turns in a coil. The three materials under test for the new set of coils are air core, aluminum core, and steel core.

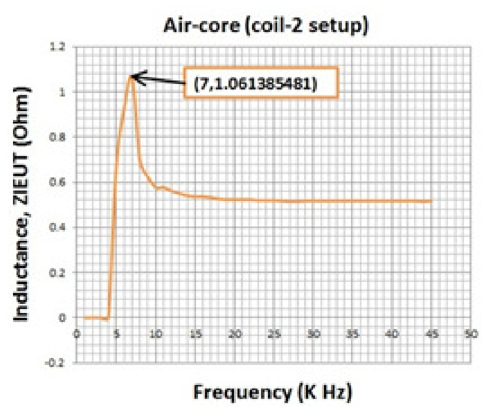

Fig. 8: Inductive peak against frequency variation for air-core coil-2 setup.

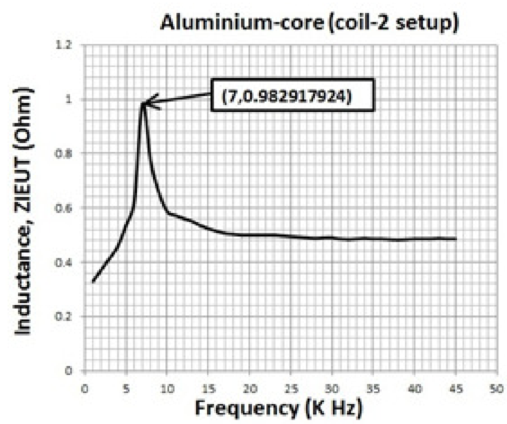

Fig. 9: Inductive peak against frequency variation for aluminium-core coil-2 setup.

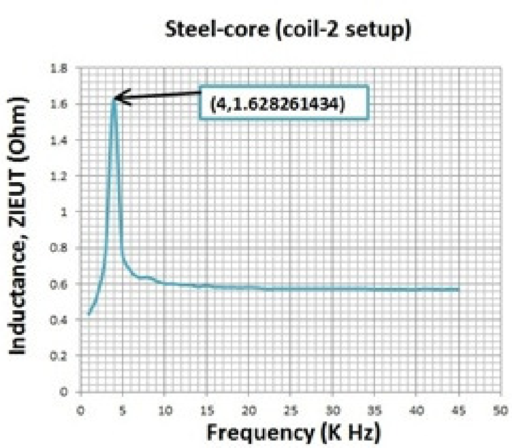

Fig. 10: Inductive peak against frequency variation for steel-core coil-2 setup.

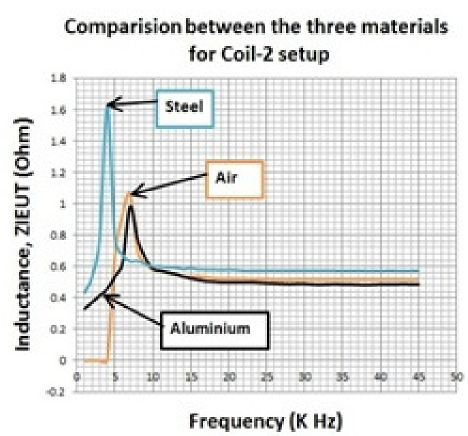

Fig. 11: Overall performance for the three materials with coil-2setup.

Figure 8 shows the inductive peak plot for air core when no material is inserted in the solenoid coil. Again the highest peak in impedance value for air core is achieved at 7 kHz, giving a high inductive peak at 1.061385 Ω. Figure 9 shows the inductive peak obtained for aluminium in coil-2 setup; aluminium was used instead of plastic in order to consider the inductive response for a paramagnetic material (7 kHz, 0.982917 Ω). Moreover not much difference was observed in the inductive peak values, because the permeability of aluminium and plastic are relatively equal. In Fig.10 the inductive peak was obtained in the case of steel core. The inductive peak was observed at a frequency of 3 kHz with an inductance value of 1.026549297 Ω.It is noteworthy that the peak value was obtained at a faster frequency response when compared with the results of coil-1 setup. Figure11 illustrates the overall performance of the materials with a higher number of turns in a coil. For air core, the highest peak is achieved once again owing to have no interruption in the electromagnetic field, as stated earlier. However, when an aluminium core was used, a lower peak value was achieved compared to steel core. This means that soft magnetic material produces a stronger magnetism compared to air and aluminium. For coil-2 setup, the number of turns was doubled, hence the magnetic field produced by each turn in the solenoid adds up, giving a stronger resultant magnetic field inside the solenoid coil. With the increased number of turns in the coil, higher voltage is induced in the coils, thus causing the inductive peak response to be faster than coil-1 setup at a lower frequency.

With the experimental setup for resistive material under test coil-1 setup had been used with a resistive value of 30 Ω and 240 Ω. Figures 12-15 present the experimental results when conducted with 30 Ω load. Figure 12 illustrates the resulting plots for paper core. The graph was plotted for impedance, Z_{MUT} as a function of frequency, which ranges from 10 kHz to 350 kHz, by an increment of 10 kHz. Paper core was used as a reference material under test. Note that there is constant increase in the graph, till 2 kHz, it then shows a continuous decrease in the values with increasing frequency. Figure 13 shows the impedance response for the case of aluminum. The nonmagnetic materials, aluminum and paper, showed almost similar output, since their permeability is approximately equal to of air. The graph showed shows a linear increment for a small change in frequency range. Moreover there was a constant decrement of the Z_{MUT} values with increasing frequency.

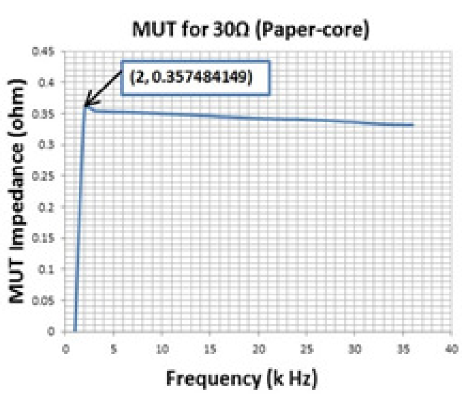

Fig. 12: Resistive materials under test with a load resistance of 30 Ω for paper-core.

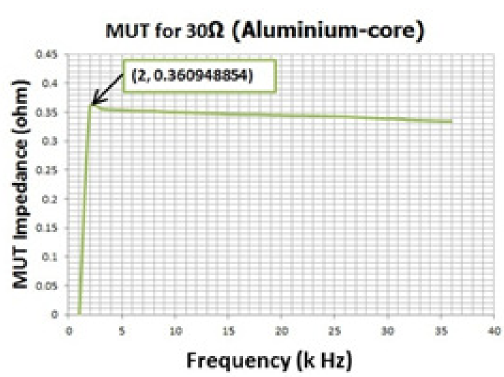

Fig. 13: Resistive materials under test with a load resistance of 30 Ω for aluminium-core.

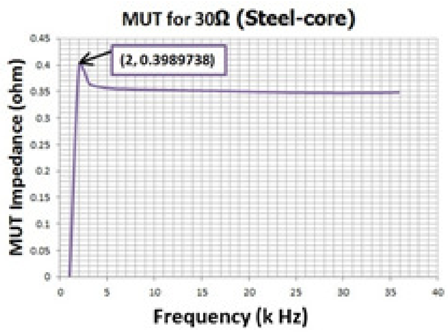

Fig. 14: Resistive materials under test with a load resistance of 30 Ω for steel-core.

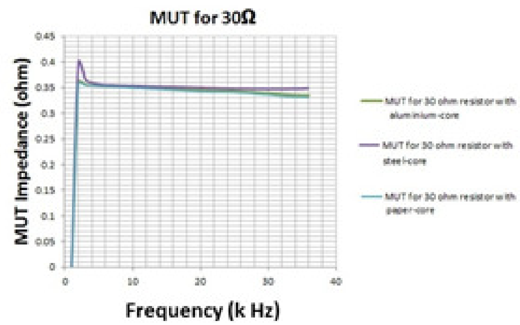

Fig. 15: Overall performance for the three materials with a load resistance of30 Ω.

Figure 14 shows the impedance response obtained in the case of steel core. Steel exhibited a relatively different plot compared to paper and aluminum, as steel is a soft magnetic material. However, atthe frequency of 2 kHz, highest impedance peak was obtained for steel. Figure 15 illustrates the results of the comparison made between the three materials under test using a 30 Ω resistor. There is an overlap in the results for paper and aluminum however there is a slight variation in the plot response for steel. The results obtained using 30Ω resistor did not show a positive outcome for the purpose of characterizing materials; hence the experiment was repeated with a higher resistance load.

Figure 16 shows the results for resistive material under test with a load resistance of 240Ω. In the case of paper core a similar output response as in the case of 30 Ω resistor was obtained. The impedance peak had resulted at a frequency of 2 kHz.

Figure 17 shows the results obtained for aluminum core condition, by using a higher resistance a very small change in the impedance peak is noted, but the frequency at which the peak occurs remains the same.

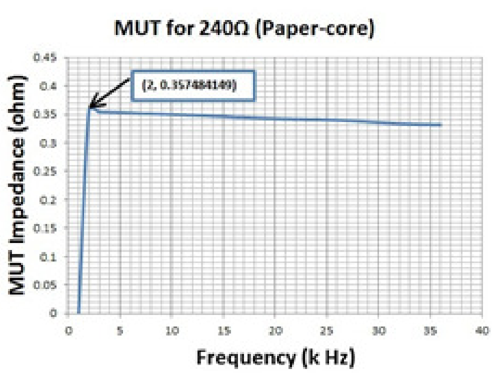

Fig. 16: Resistive materials under test with a load resistance of 240 Ω for paper-core.

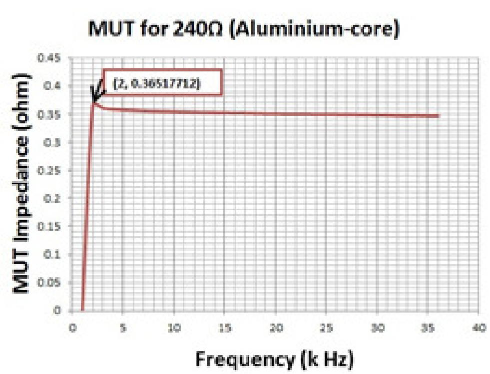

Fig. 17: Resistive materials under test with a load resistance of 240 Ω for aluminium-core.

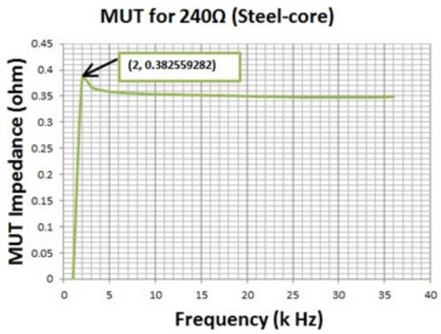

Fig. 18: Resistive materials under test with a load resistance of 240Ω for steel-core.

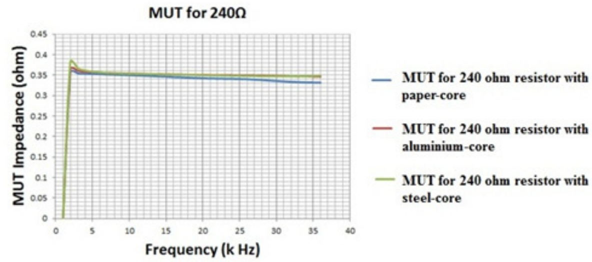

Fig. 19: Overall performance for the three materials with a load resistance of 240Ω.

Figure 18 shows the impedance response obtained for the case of steel core. Steel exhibited a higher impedance peak compared to paper and aluminium. Figure 19 illustrates the results for the comparison of the three materials under test with load resistance of 240 Ω. Note that not much difference was observed between the high load resistor and the low load resistor for the three resistive materials under test. The results almost overlap. This shows that load resistance in the main circuit is not affecting the current induction on the coil for this specific material; therefore resistors are not very applicable to be used in transducers. Moreover it can be concluded that in order to characterize materials, inductors are more well-suited than resistors.

As a whole, the distinctive impedance peaks are basically representing the core materials. Referring to Fig.11, impedance peaks are shown for three materials, i.e., steel, air and aluminium. The figure apparently shows that aluminium and air (both non-magnetic) are very close to each other in their magnetic properties by exhibiting their peaks at almost the same frequency between 5 and 10 kHz, whereasbeing magnetic steel shows similar peak at a frequency of about 0.9 kHz, showing resonating behaviour at a much lower frequency. The materials are characterised for their magnetic behaviours at low frequency, justifying the characterization technique accordingly.

6. CONCLUSION

In this paper, a magneto-inductive sensor model is used in which magnetic and non-magnetic materials are inserted to characterize the material by measuring the inductance of magnetic and non-magnetic inductance peaks. The characteristic of the material can be differentiated from other materials by obtaining unique distinctive impedance peaks with unique impedance values. The change in permeability of a material determines the inductance of the output coil, which is changed by changing the positioning of movable core made of the material. This paper uses a simple circuit approach by making use of inductive coupling approach. This characterization technique of magnetic and nonmagnetic material is significantly important for the applications in biomedical equipment and implants requiring low frequency. Another reason for this identification technique also comes from the particular fact that magnetic materials over the last twenty years have changed dramatically both in terms of their applications and in the magnitude of their use.

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NOMENCLATURE

A	Area of coil	m^2
l	Length of the coil	m
l-x	Core displacement left from the left end	m
L	Total inductance	H
L_0	Inductance of free space	H
N	Total number of turns	-
x	Core displacement fron the right end	m

GREEK LETTERS

μ_o	Free space permeability	Hm^{-1}
μ_r	Permeability of a material	Hm^{-1}