EXPERIMENTAL STUDIES ON THE QUASI-STATIC AXIAL CRUSHING BEHAVIOR OF FOAM-FILLED STEEL EXTRUSION TUBES

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ABSTRACT: The concerns of automotive safety have been given special attention in order to reduce human fatalities or injuries. One of the techniques to reduce collision impact or compression energy is by filling polymeric foam into metallic tubes. In this work, polyurethane foam was introduced into the steel extrusion tubes and quasistatically compressed at constant cross-head displacement. Different tube thicknesses and foam densities were used and these variables were related to the crashworthiness aspect of the foam-filled structures. It is found that both tube thickness and foam density played an important role in increasing the crashworthiness behaviors of the structures but when the tube thickness reached a certain value, foam density reduced the energy absorption performance of the structures.

WORDS: Foam-filled structures, Axial crushing, Energy absorption, Steel extrusion tubes.

1. INTRODUCTION

The emphasis on safety in automobiles has increased the need for more information on energy absorbing materials in order to protect a driver from injury in a traffic accident. The compressive behavior of a material should be ingeniously utilized for design of energy absorbing structures [1]. Vehicle size and mass provide a certain degree of protection but can have negative effects. Driven by the need to overcome these negative exists of both size and mass coupled with mandates for increased fuel efficiency, an attempt is being made to use foam-filled structures in the development of energy dissipating devices. The challenge is the use of specific features of geometry and materials meabling greater safety while simultaneously decreasing the weight, without negatively affecting the overall economic of fabrication and production [2]. Particular interest to this state use of structural foams in automotive components. Foam is currently being as a filler material in bumper and as reinforcement in roof and door beams. Foam has the subject of numerous experimental and numerical and theoretical investigations. Example 2 al. [3] emphasize the integration of foam materials in the automotive body structure for energy absorption. Sugimura et al. [4] and Grenestedt [5] assessed the role of morphology and imperfections in governing the basic properties of foams such as simess, yield strength and fracture resistance. Ford and Gibson [6] developed microstructural models to examine the mechanisms responsible for differences in tensile and compressive strength observed in cellular materials. Cheon and Meguid [7] proposed a

modified and representative unit cell model employed to study the crush behavior of closed cell foam. Currently, research into modelling the mechanical response of cellular solids is progressing on two fronts, i.e. phenomenological models and unit-cell based models. Gibson and co-workers [8] developed a skeletal unit-cell to model open-cell foams and implemented dimensional arguments to relate the strength and elastic constants of the unit-cell to its relative density. They extended this model to closed-cell foam and used it to identify the prominent modes of failure under multiaxial loads. Santonsa and Wierzbicki [9] developed a closed cell model for metallic foams, based on careful examination of the foam morphology. The theoretical solution was in agreement with the experimental findings and finite element simulations. Meguid et al. [10] modified the model in [9] to better reflect the morphological features of closed-cell aluminium foams, and implemented it to study the effects of three dimensional density gradients on the deformation patterns, crush behavior, and energy absorption characteristics. The localization patterns and the normalised crush load-deformation curves were compared with in-plane and transverse crushing experiments and found to be in good agreement. Alexander [11], and Pugsley and Macaulay [12] presented the earliest analytical treatment of the quasi-static axial crushing of thin-walled cylinders for both symmetric and asymmetric modes of collapse. Their treatment is based on an idealised folding mechanism and rigid plastic material model and the work component necessary to form plastic hinges in the cylinder. Abramowicz and Jones [13, 14] enhanced Alexander's solution by considering the incremental variation in dissipated energy with varying circumferential strain rather than considering its mean value. They found reasonable agreement between the analytical results and experimental findings under quasi-static test conditions for a wide range of diameter-to-thickness ratios. Wierzbicki and Bhat [15] developed a moving hinge solution for the axisymmetric collapse mode during axial crushing of cylindrical tubes. This model allows the prediction of the complete loaddisplacement history. Grzebieta [16] implemented a strip method and the moving hinge solution to evaluate load displacement relations for both axisymmetric and asymmetric collapse modes and found good agreement between analytical and experimental results under quasi-static and dynamic conditions.

L. Altay et al. [17] found the number of folds formed in foam-filled tubes, both in diamond and concertina mode of deformation, increased with foam filling and also with increasing foam filler density. It was further found that the restraining effect of filler shifted the deformation mode from diamond to concertina in the larger diameter tube investigated. Similar results were obtained numerically where the energy absorption in foam-filled were shown to increase with increasing filler density and higher than the sum of the energy absorption of empty and foam-filled tubes. While Goangseup Zi et al. [18] experimentally studied on the bridge deck filled with polyurethane foam under quasi-static loading. The results of study found that the structural properties improved significantly by using the low density foam. For this purpose of this study, relatively high and low density foams of polystyrene closed cell foams were used to fill a square thin-walled steel extrusion tube with different wall thicknesses. The energy absorption in foam filled tubes was then determined as functions of foam densities and wall thicknesses. The collapse mechanisms of the empty and foam-filled tubes were observed and analyzed. In this study, an experimental work has been conducted to study the crushing behavior of the foamfilled steel extrusion under quasi-static compression loadings. Two main variables were used such as wall thicknesses and foam densities. Force versus displacement diagrams of each test were analyzed and discussed, then several crashworthiness aspects such as

energy absorption performances, mean forces and peak forces were studied. Finally, the results from these tests were related to the mechanisms of the structural collapses.

2. MATERIAL AND TESTS

2.1 Polymeric Foam and Foam-Filled Steel Extrusion Preparations

The deep drawn mild steel tube studied was 50x50x200 mm with a wall thickness of 1.0, 1.5, 2.0 and 3.0 mm was used in this study. The tube shape is shown in Fig. 1. The chosen tube lengths were mainly dictated to avoid buckling behavior which would result in lower energy absorption. Polyurethane foam was prepared using Polyol and Isocynate in the liquid forms. The foam liquids (Polyol and Isocynate) were mixed using 100:110 ratio and the liquid mixtures were poured into steel extrusion after mechanically stirred for 0.5 minutes at low stirring speed (20 rpm) in order to have homogenous cellular materials. Three different densities were prepared; 100, 200 and 300 kg m⁻³. The foam-filled structure produced is shown in Fig. 1b.





Fig. 1: (a) Empty and (b) foam-filled steel extrusions.

2.2 Microstructure and Mechanical Behavior

The microstructures of the foams were also observed for each foam density. It is very crucial to study the influence of the foam morphology on the energy absorption capability. Scanning electron microscope (SEM) was used to observe the microstructures. Before the observation is conducted, the surface of the foam must be coated by a layer of gold. This coating is to prevent electro discharge which can be interrupted in the observation process.

2.3 Crushing Tests

Quasi-static compression tests on foam samples, empty and foam-filled tubes were conducted using a controlled displacement with a SHIMADZU AG-I universal testing machine with a displacement rate of 1.5mm/min. Force-displacement curves for each sample were recorded automatically. The corresponding average crushing loads (Pa) of the tested tubes were calculated using Eq. (1.0). This means that the load fluctuation during the crushing process was smoothened by summing minimum and maximum loads and dividing with the number of total points of the considered values.

$$P_a = \frac{\int Pd\delta}{\delta} \tag{1.0}$$

where P, P_a and δ are the applied load, mean load and displacement, respectively. The area under the curve also represents the energy absorbed during the crushing process, E. It was calculated by multiplying average crushing load with crushed distances. The energy absorption performance, E can also be calculated using Eq. (2.0).

$$E = \int P ds \tag{2.0}$$

where s is the distance along the force-displacement curve and ds is the small element on that curve.

2.3 Crushing Observations

In this observation, only final collapse mechanisms are presented to study the influence of tubes geometries and foam densities on the quasi-static energy absorption performance. The most important parameter in this observation is the formation of folding pattern with different wave length. The difference in the wave length of folding pattern determines the capability of the structures to absorb the crushed energy.

3. RESULTS AND DISCUSSION

3.1 Foam Microstructures

Many cellular solids are excellent energy absorbers owing to their deformation at a nearly constant level over a wide range of strain [1-5]. Due to high porosity, foam in general can undergo large inelastic strain that include an initial linear elastic behavior, followed by a plasticity-like stress plateau where the foam cell starts to crush and finally a steep densification regime where the foam densifies rapidly. Therefore, the observation of foam morphology is essential and important to study the effect of foam geometry on the energy absorption performances. Figure 2 shows the foam morphology for different foam densities. From the SEM observations, it is found that when the foam density increases, the size of the foam is increased and the shape of the foam also changes from irregular to regular geometries. All the foam morphologies are categorized as closed-foam cell. It can be predicted that the higher the porosity level of the foam, the more strain the specimen can accumulate before being fully densified during the compression. On the other hand, the irregular shape of the foam capable to initiate the localized plasticity at the hinge of the cell wall and this condition facilitates the localized bending the cell and therefore affected to the whole structures. This phenomenon results the crushed foam reaches saturation phase quicker and therefore, the performance of energy absorber is lowered. In comparisons, the cell wall is very thin and the ridge is also thin and rather straight for 300 kg/m3 foam density compared to others.

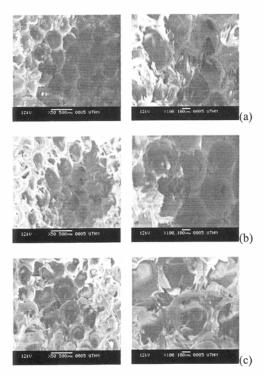


Fig. 2: The microstructures of the foams at different foam densities, (a) 100 kg m⁻³, (b) 200 kg m⁻³ and (c) 300 kg m⁻³.

3.2 Compression Behavior of the Foams

The compressive response of the foam can be found in Fig. 3. All of the foams showed a similar compressive response exhibiting three distinct regions. The first regime is a linear elastic response, where the stress is proportional to the strain, the second regime is a plateau region where the stress is constant while the strain increased and the final regime is a nonlinear state where the stress increases significantly for a small increase in the strain of the foam due to the densification of cellular material where the cells have totally collapsed and the cellular walls come into contact. The compressed foams indicated that the plateau stress increased with increasing the foam density. The strain at which the densification starts decreases with increasing foam density. This is also related to the number of cells wall collapse. As mentioned earlier, the 300 kg m⁻³ foam density accumulated higher yield and plateau forces. Figure 3 also agreed with the foam morphology as shown in Fig. 2.

0.5

0.01

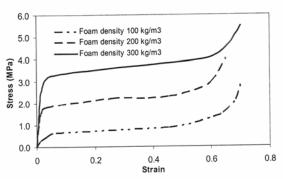


Fig. 3: The response of polymeric foam of different densities under compressive loadings.

The plateau stresses are determined from the initial flat regions of the curves as marked in Fig. 3 and the corresponding average plateau stress values of polymeric foams are listed in Table 1. The plateau stress, σ is found to be well fitted with power-law of strengthening Eq. (3.0) as shown in Fig. 4:

$$\sigma = K\rho^n \tag{3.0}$$

Where K and n are constants and ρ is the foam density in k gm⁻³. The values of K and n are 0.8718 (MPa) and 1.3141, respectively.

Table 1: Average plateau stress values of polymeric foams.

Foam density (k gm ⁻³)	Plateau stress (MPa)		
100	0.87		
200	2.18		
300	3.68		
4.5			
4.0	0.0749/donoity)1.3141 I		
ॡ 3.5 -	stress = 0.8718(density) ^{1.3141}		
(8) 3.5 - W) 3.0 -			
% 2.5 -			
₩ 2.0 -			
so 2.5 - 1.5			
<u>a</u> 1.0 -			

From density (g/cm3) $Fig. \ 4: The \ relationship \ between \ plateau \ stresses \ to \ foam \ density.$

0.02

0.03

The energy absorbed by the foams are determined from Fig. 3 and presented in Fig. 5 for each foam density. It is shown that the densities play an important role in increasing the performance of energy absorption. High porosity content controlled the plastic collapse of the foam wall.

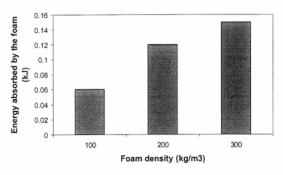


Fig. 5: The effect of foam density on the energy absorption capability.

3.3 Effect of Thickness on the Force-Displacement Curves.

The typical force-displacement curves for empty and foam-filled steel extrusion tubes are presented in Fig. 6-9 for all types of tube conditions. Three phases can be observed in these diagrams such as linear elastic deformation, load fluctuation and densification stages. For a given type of material, the differences in the load-displacement curves mainly occurred in all phases. Significant offset occurred at the beginning of the crushing processes. A 3 mm thick wall tube exhibited greater peak and mean forces compared to other type of tube wall thicknesses. It is also shown that large force drop occurred just after the linear elastic deformation. The force drop is always greater than that other type of foam-filled tubes. This is due to the highly localized plasticity towards buckling behavior and strongly related to the formation of the wall folding. During crushing, the top end of the tubes experienced high compression force and initiated the plastic deformation enough to fold the tube wall. Then, the first lobe started to appear after localized buckling of the wall and progressively collapse downward to the bottom end of the tubes. According to Fig. 6-9, the thicknesses lower than 2 mm produced lower peak and mean forces compared to the same conditions obtained from 3 mm thick wall tubes and the response of the forcedisplacement diagrams are not significant when filling with higher foam density. This is mainly due to the change of the wall thickness. For 3 mm thick walls, strains along the wall constraint the deformation of the tubes. Therefore, this mechanism produced larger force-displacement curves compared to other type of tubes.

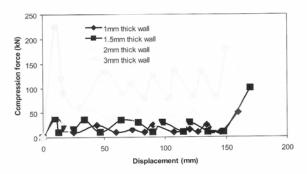


Fig. 6: The force-displacement curves of empty steel extrusions.

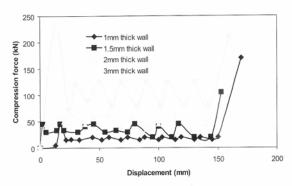


Fig. 7: The force-displacement curves of 100 kg m^{-3} foam filled steel extrusions.

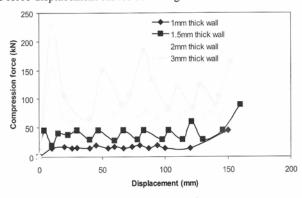


Fig. 8: The force-displacement curves of 200 kg $\,\mathrm{m}^{-3}$ foam filled steel extrusions.

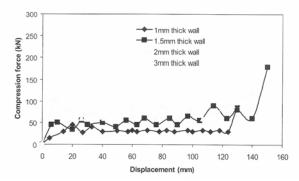


Fig. 9: The force-displacement curves of 300 kgm⁻³ foam filled steel extrusions.

3.4 Effect of Foam Density on the Force-Displacement Curves.

Figures 10-13 present the force-displacement curves of the empty and foam-filled tubes constructed of different tube thicknesses. The purposes of these diagrams are to reveal the effect of the foam density filled into the tube. It is because the foam is unable to push the tube wall circumferentially and therefore, cellular foam plastically deformed under high compression pressures. The effect of foam density becomes more obvious when 300 kg/m³ foam density is used where the mean force of the fluctuated forces increased. This behavior shows that the capability of this tube to absorb more compression energy compared to low density foam. When the foam density is increased, the forcedisplacement diagrams changed obviously but the presence of the foam do not affected significantly when 3 mm thick wall is used. This is due to the fact that the foam itself can not sustain higher compression load created by this type of wall thickness. Regarding to Fig. 13, the force-displacement curves for all type of the tube conditions is very similar to each other meaning that both peak and mean forces and other crashworthiness behaviors are not greatly affected by the foam even when higher density foam is used. For the tube wall less than 2 mm, the effect of the foam density is significant. But there is unclear relation between foam densities and the compression forces for different wall thicknesses. Figure 13 reveals that the foam densities not a key factor to increase the response of the force-displacement curves.

3.5 Effect of Thickness on the Peak Force, Mean Force and Force Ratio.

Figures 14-16 summarize the crashworthiness behavior of the empty and foam-filled tubes compressed quasi-statically. These figures show that the peak and mean forces increased when the wall thickness increased. It is also shown that both forces are strongly dependent on the foam density and they are always higher than that of empty tubes. For 1.0 mm thick wall tube, small forces variation occurred showing that it is independent on the foam density. This is may be due to the effect of the foam densification when the tube progressively crushed. The densification of the foam forced the wall outward of the tube and therefore reducing the tube strength and integrity. When the wall thickness and foam density increased, the peak force is also gradually increased as a result of increasing of the cross-sectional area of the tube and higher foam density resisted the elastic and plastic

deformation of the foam-filled tubes. Figure 15 reveals that, high force ratio occurred for tubes filled with 300 kg m⁻³ foam density. Large force ratio indicated that high tendency of those tubes to fail catastrophically. If the tube fails catastrophically, this will result in lower energy absorption capability. This phenomenon occurred especially for thicknesses less than 1.5 mm due to high pressure created by the foam during deformation. This pressure or hoop stress of the foam acted circumferentially, that forces on the tube wall and it deformed the wall prematurely. But when the wall thickness is increased, the force ratio is reduced significantly. It is shown that when 3.0 mm thick wall is used, foam of different densities do not contributed to increase the energy absorption of the tubes. This is because the strength of the densified foam unable to deform the wall tubes even high density foam was used.

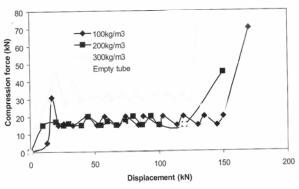


Fig. 10: The force-displacement curves of the steel extrusion tubes of 1.0 mm wall thickness filled with different foam densities.

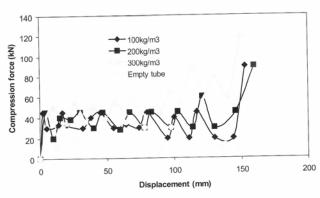


Fig. 11: The force-displacement curves of the steel extrusion tubes 1.5 mm wall thickness filled with different foam densities.

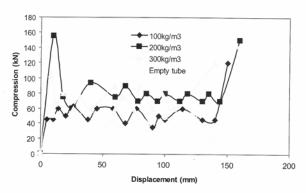


Fig. 12: The force-displacement curves of the steel extrusion tubes of 2.0 mm wall thickness filled with different foam densities.

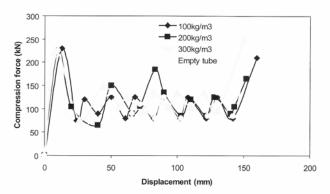


Fig. 13: The force-displacement curves of the steel extrusion tubes of 3.0 mm wall thickness filled with different foam densities.

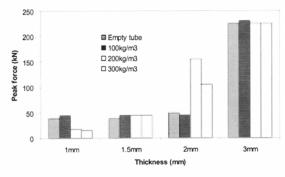


Fig. 14: The effect of wall thickness on the peak force.

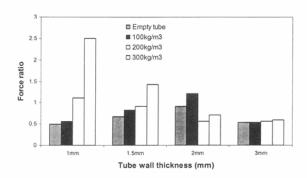


Fig. 15: The effect of wall thickness on the force ratio.

3.6 Effect of Wall Thickness on the Energy Absorption Capabilities.

The effect of wall thickness and the peak force on the energy absorption performances is presented in Fig. 16 and 17, respectively. It is shown that for 1.0 mm tube thickness, increasing the foam density do not affected the capability of the energy absorption significantly. Small energy absorption variation can be seen and this effect is almost can be neglected. When the tube thickness increased, the influence of foam density is significant where the energy absorbed by the tubes increased especially for thickness. Foam does play an important role in improving the capability of steel tubes to absorb the compression energy. Significant improvement can be seen for tube thicknesses range between 1.0 to 2.0 mm where the energy absorption increased when foam density increased. The influence of the foam filled into 3.0 mm tube thickness is not obvious. It seems that the foam do not affected the energy absorption capability of the tubes.

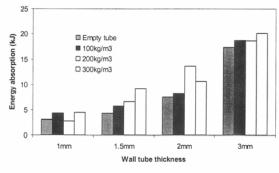


Fig. 16: The effect of wall thicknesses on the energy absorption capabilities.

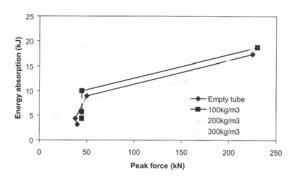


Fig. 17: The effect of aspect ratio on the energy absorption capabilities.

3.7 Effect of Foam Density on the Energy Absorption Capability.

The effect of foam densities on the energy absorption for different wall thickness is presented in Fig. 18. It is clearly stated that the energy absorption increased slightly with increasing densities. Surprisingly, thicker steel extrusion tubes demonstrated an improvement in energy absorption performances but the increment is not significant when the foam density is increased. Figure 18 also reveals that the energy absorption for 2.0 mm thick tube reached to the same value as 3.0 mm thick tube when filling with 300kg/m^3 and for other tubes which the thickness less than 2.0 mm, the tendency of the capability to absorb the compression energy is appropriate when higher foam density is used.

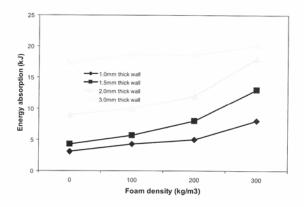


Fig. 18: The effect of foam density on the energy absorption capabilities for different wall thicknesses.

3.8 Collapse Mechanisms of the Empty and Foam-Filled Tubes.

It was experimentally found that empty and foam-filled tubes progressively collapse and the respective force-displacement diagrams for each tube conditions were presented in Fig. 7-10. Figure 20 reveals the final deformed tubes of empty and foam-filled tubes with different foam densities and wall tube thicknesses. The folding patterns of the deformed tubes are of symmetrical shapes with extensional folding modes with all the lobes moving outwards. Large hoop strains occurred around the tube wall during the localized buckling. Also, Fig. 20 shows that there are two different behaviors are observed; the number of lobes of folding and the thickness or length of the lobes. These factors contributed to improve the crashworthiness behaviors of the tubes. Higher crashworthiness behaviors are obtained when both wall thickness and foam density is increased. The number of lobes is related to the plastically deformed of tube wall. During the deformations, the energy dissipation increased and therefore lowering the energy absorbed by the tubes. For example, 1.0 mm wall tube thickness has five numbers of lobes resulting in lower energy absorption capability compared to other tubes which having in averagely 4 number of lobes and therefore sustained higher energy absorption. The second factor is the thickness of the individual lobe. For foam-filled tubes, the thickness of the lobes is thicker than unfilled tubes. This is due to the flow of the foam where during the crushing process, the volume of the foam change where the lateral expansion of the foam entered the new space created during the formation of the first lobe outwards of the wall. The extremely densified foam occupied this newly formed space tightly so that no subsequent deformation of the lobes occurred and contradictorily behavior is obviously observed for empty tubes.

Foam	Tube wall thickness (mm)				
density	1.0	1.5	2.0	3.0	
Empty tube		(pade)			
100 kg m ⁻³					
200 kg m ⁻³	1				
300 kg m ⁻³					

Fig. 20: The effect of foam densities and wall thicknesses on the final progressive collapses of the steel tube extrusions.

4.9 Foam-Wall Interactions

The interaction between foam and tube wall was clearly revealed through Fig. 21 and the respective collapse mechanisms. The interaction between the tube and foam are also presented in Fig. 22. Force-displacement diagram for each single element in the foamfilled columns showing the important of utilizing the polymeric foam filled into the steel tube extrusion. Figure 21 shows that the response of foam alone is insignificant to be used as an energy absorbing device. Hanssen et al. [19] and H. W. Song et al. [20] described the interaction effect as the following: the increased number of lobes created by introducing foam filler causes the force level of the foam-filled columns to be significantly higher than that of the combined effect of non-filled column and foam alone and similarly, the same interaction effect is prominent in the foam-filled hat sections in the present study. Figure 21 has illustrated the interaction effect in the form of crushing force histories. Foam-filled column shows higher force versus displacement response. This effect is due to the foam densifications especially for the foam adjacent to the wall. During the localized plastic buckling of the wall or so-called the lobes, the densified foam entered into the lobes progressively and prevented the lobes from fully deforming. This mechanism provided a strengthening effect of the tube because the space of the lobe occupied by the extremely densified foam. The characteristic of the foam is illustrated by Hong-Wei Song et al. [20].

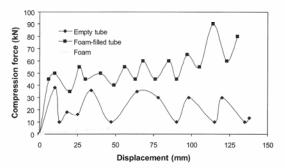


Fig. 21: The force-displacement shows the interaction between foam and wall under compression forces.

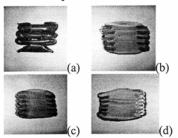


Fig. 22: The sectioned final deformed showing the interaction between foam and wall, (a) empty tube, (b) 100 kg m⁻³ foam density, (c) 200 kg m⁻³ foam density and (d) 300 kg m⁻³ foam density filled steel tubes.

4. CONCLUSION

The current study aimed at investigating the crush behavior of polymeric foam-filled steel tube extrusions, while the foam filler was polyurethane. The tube wall thicknesses and foam densities were varied in order to study the crushing behavior and the capability of the energy absorbed by the tubes. It was found that when both wall thickness and foam density increased, the crashworthiness behaviors of the tubes also increased gradually. But when the wall reached 3.0 mm thick, foam does not play an important role in improving the crashworthiness performances. From the observation of the crushed behaviors, two distinct things occurred; (1) the difference in the number of lobes or folding of the wall, and (2) different in the length of the individual lobe. These phenomena contributed to changing the crashworthiness capability due to the interaction between foam and tube wall.

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